Using ICT to Support RME-Based Mathematics Teaching and Learning at Junior Secondary School

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A. Introduction

The development and availability of information and communication technology (ICT) – including computer, multimedia, and the Internet – has been providing new, alternative, ways of teaching, learning, and doing mathematics. The integration of ICT into mathematics curriculum for schools has also been initiated in many countries, including in Southeast Asia countries, like Indonesia, Singapore, and others. During the last decades, the use of ICT tools for teaching and learning in mathematics classrooms has been receiving a lot of attention. It is believed that the integration of ICT into mathematics curriculum will give some benefits, such as: (1) ICT enables to visualize and simulate the real situations in an instant way, (2) it supports the understanding of mathematical concepts, because many abstract mathematical concepts can be visualized and simulated, and (3) it helps students investigating and exploring mathematical problems and checking hypothesis or conjectures. Because of its benefit, ICT can be used to support an activity-based approach or a realistic approach to mathematics instruction, such as realistic mathematics education (RME).

According to RME approach, which is based on the view of Freudenthal (1991), mathematics is considered as a human activity and it must be connected to reality (real world or situation which is real in students' mind such as application or modeling), which is used as a source for mathematization (Zulkardi, 2010: 3). Furthermore, Zulkardi explained that because mathematics is considered as a human activity, mathematics education is organized as a process of *guided* reinvention (instructional environment that provided steps of the learning process), by which students can experience a similar process to the invention process of mathematics. Referred to Treffers (1987) and Freudenthal (1991), Zulkardi also explained the two types of mathematization activities:

1. **horizontal mathematization** (going from real world to symbols, the students apply mathematical tools to organize and solve a problem located in a real-life situation): identifying or describing the specific mathematics in a general context, schematizing, formulating and visualizing a problem in different ways, discovering relations, discovering regularities, recognizing isomorphic aspect in different problems, transferring a real world problem to a mathematical problem, and transferring a real world problem to a known mathematical problem; and
2. **vertical mathematization** (moving within the world of symbols, the process of reorganization within the mathematical system itself): representing a relation in a formula, proving regularities, refining and adjusting models, using different models, combining and integrating models, formulating a mathematical model, and generalizing.

In addition, quoting Streefland (1991), Zulkardi also explained that RME-based teaching and learning has four characteristics: (1) situating the intended material in reality, both as source well as area of application, starting from meaningful contexts having the potential to produce mathematical material involves (horizontal mathematization); (2) intertwining with other strands, such as fractions and proportions, functions and curves, etc.; (3) producing tools in the form of symbols, diagrams and situation or context models during the learning process through collective effort (vertical

mathematization); and (4) learning through constructions is carried out by arrangements of the students activities, so they can interact with each other, discuss, negotiate, and collaborate.

In supporting RME-based lessons, ICT can be used during either horizontal or vertical mathematization processes, such to visualize a problem in different ways, to explore, recognize, and discover relations and patterns, and to refine and adjust models. ICT can be used also to help students doing activities and interactions with other students.

There are different ICT tools that can be used to support RME-based teaching and learning mathematics for Junior High School's students. Among them are the Internet resources, multimedia tools, electronic spreadsheets, and dynamic mathematics software. This paper will focus on the explanation of the use of the Internet resources, electronic spreadsheets, and some dynamic mathematics software to support RME-based mathematics instruction in Junior High School.

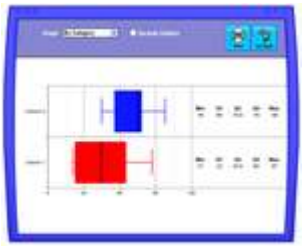
B. Using Internet

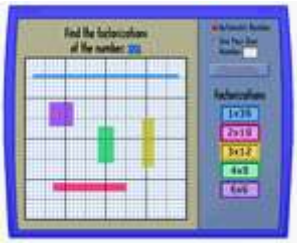
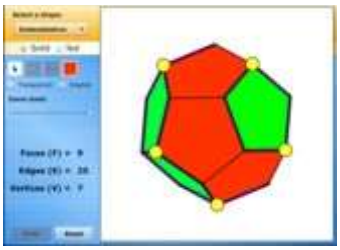
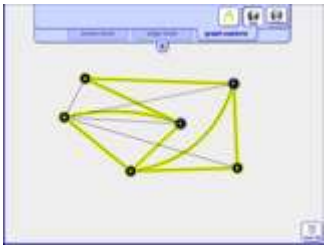
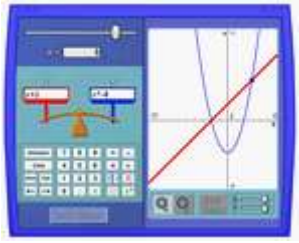
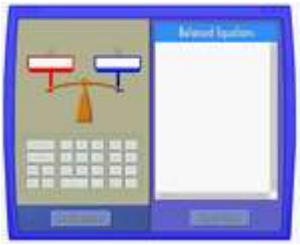
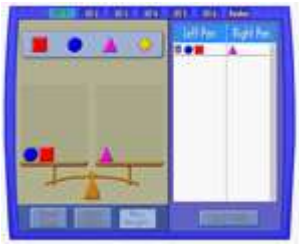
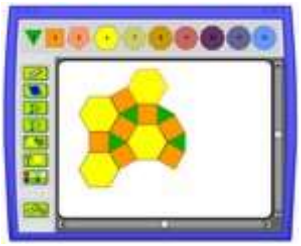
Internet provides many resources for teaching and learning mathematics for Junior High Schools. In the Internet we can search and find many teaching and learning materials for special topics in mathematics. Many research institutes, educational software developers, professional organizations, and individual professionals developed Web sites that provide teaching and learning resources for mathematics instructions. The following are some useful Web sites that can be used by teachers and students to teach and to learn mathematics.

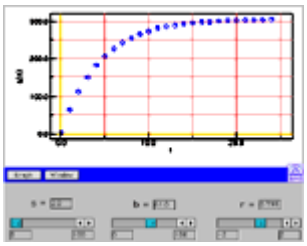
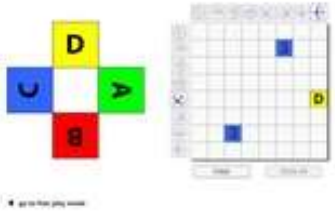
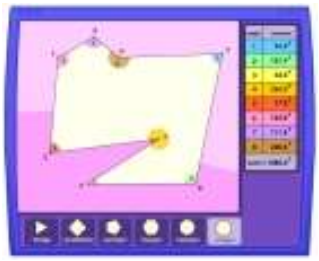
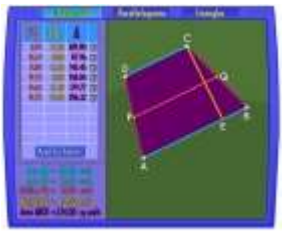
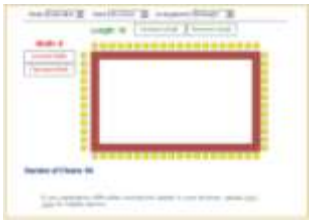
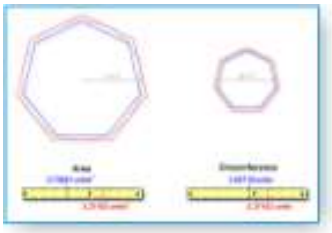
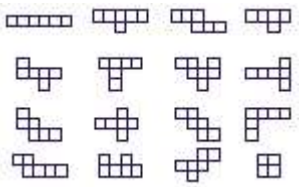
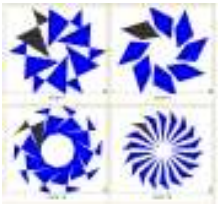
1. Illumination by NCTM (http://illuminations.nctm.org/)

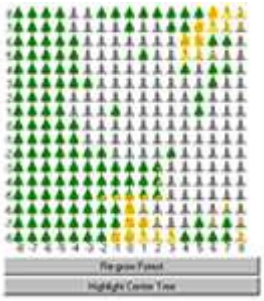
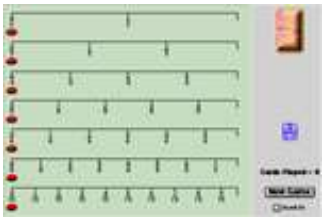
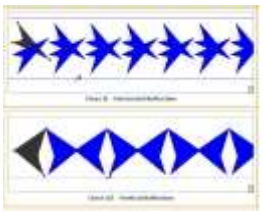
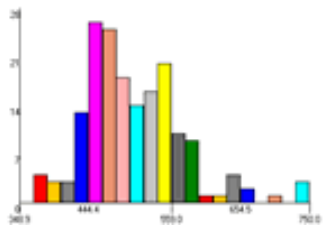
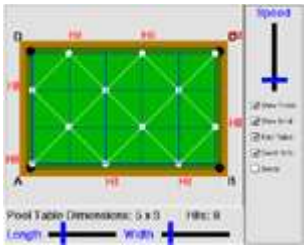
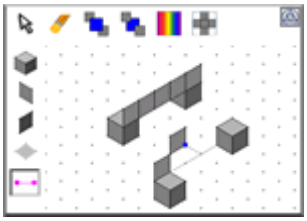
Illumination is part of NCTM Web site that is dedicated to provide resources for teaching and learning mathematics for pre-K12. The Web site provides information on pre-K12 Mathematics Standards, collections of Lessons, and Collections of Activities that can be accessed freely by teachers and students. There are 108 interactive activities, categorized based on student's grade: pre-K2, Grade 3 – 5, Grade 6-8, and Grade 9 – 12. The activities cover many topics for pre-K12 mathematics. On each activity, there is an explanation about the activity and what students have to do and an applet by which students can do activities interactively.

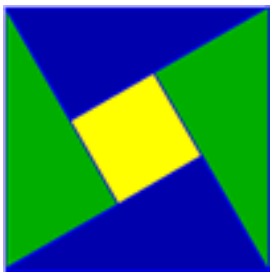
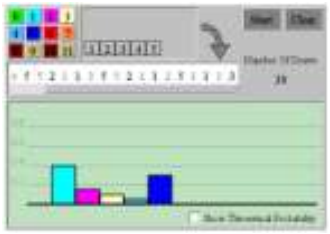
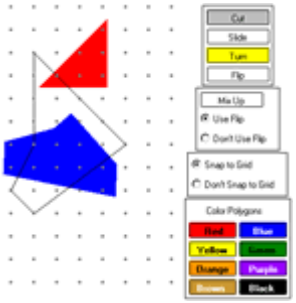
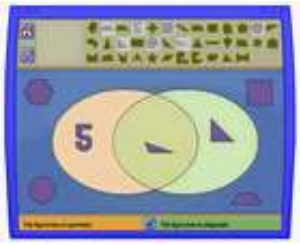
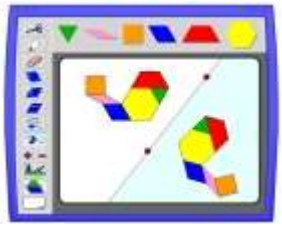
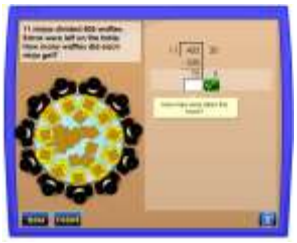
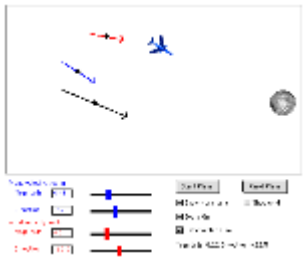
The following are part of the collection of activities appropriate for grade 6 – 8 (Junior High School).

Advanced Data Grapher	**Algebra Tiles**	**Deep Sea Duel**
		
Analyzing Data with Box Plots, Bubble Graphs, Scatter plots, Histograms, and Stem-and-Leaf Plots	Manipulating Algebra Tiles to Solve Equations, Substitute in Expressions, and Expand and Factor	Playing a Strategy Game That Requires You to Select Cards with a Specified Sum Before Your Opponent

Data Grapher	**Dynamic Paper**	**Factorize**
 Analyzing Data with Bar Graphs, Line Graphs, Pie Charts, and Pictographs	 Creating Customized Activity Sheets for Your Classroom	 Dividing Numbers into Two Factors and Building Arrays to Represent Each Factorization
Fractal Tool	**Fraction Models**	**Geometric Solids**
 Exploring Iteration and Patterns in Shapes and Numbers with Fractals	 Exploring Several Different Representations for Fractions	 Manipulating Various Geometric Solids and Investigating Their Properties
Graph Creator	**Pan Balance – Expressions**	**Turtle Pond**
 Creating and Exploring Vertex-Edge Graphs	 Investigating the Concept of Equivalence by "Weighing" Numeric and Algebraic Expressions	 Estimating Length and Angle Measure While Guiding a Turtle to a Pond Using Computer Commands
Pan Balance – Numbers	**Pan Balance – Shapes**	**Tessellation Creator**
 Finding Equivalent Numerical Expressions Using a Balance Scale	 Exploring Relationships Among the Weights of Various Objects by Placing Them on Either Side of a Balance	 Creating Patterns to Cover the Screen Using Regular Polygons

Vector Investigation: Boat to the Island	**Adjustable Spinner**	**Affine Recurrence Plotter**
 Adjusting the Magnitude and Direction of a Velocity Vector to "Drive" a Boat	 Creating a Spinner to Examine Experimental and Theoretical Outcomes	 Investigating Affine Recurrence Relations of the Form $A(n) = b \times A(n-1) + c,\ A(0) = a$
Affine Recurrence Spreadsheet	**Algebraic Transformations**	**Angle Sums**
 Investigating Affine Recurrence Relations with a Preloaded Spreadsheet	 Exploring Commutativity and Associativity Within a Geometric Situation	 Exploring the Sum of the Interior Angle Measures for Various Polygons
Area Tool	**Chairs**	**Circle Tool**
 Investigating How the Base and Height of Trapezoids, Parallelograms, and Triangles Affect Area	 Exploring the Number of Chairs Needed when Tables Are Arranged in a Restaurant	 Comparing the Circumference and Area of a Circle to Its Radius and Diameter
Computing Pi	**Cube Nets**	**Cyclic Figures**
 Comparing Two Methods for Computing Pi	 Examining Various Two-dimensional Figures to Determine Which Ones Can Be Folded into a Cube	 Recognizing Rotation Symmetry in Figures and Examining Various Rotation Symmetries

Dihedral Figures	Distance to Horizon	Factor Game
 Recognizing Dihedral Symmetry and Reflections in Figures and Examining Various Symmetries	 Investigating the Relationship Between Your Elevation and the Distance You Can See to the Horizon	 Exercising Your Factoring Ability Against a Human or the Computer
Fire	**Fraction Game**	**Free Ride**
 Simulating the Spread of a Wildfire Using Probability	 Exploring Relationships Among Fractions While Playing an Interactive Game	 Exploring Fractions Using the Context of a Bicycle and Gear Ratios
Frieze Patterns	**Histogram Tool**	**Interactive Geometry Dictionary: Areas in Geometry**
 Experimenting with Frieze Patterns and the Transformations That Constitute Each of Their Categories	 Creating a Customized Histogram with Your Own Data or an Included Set of Data	 Investigating and Understanding the Area of the Rectangle, Parallelogram, and Triangle
Mixtures	**Paper Pool**	**Isometric Drawing Tool**
 Exploring Mixture Problems Using Colored Circles from Two Different Piles	 Shooting a Pool Ball to Predict Patterns	 Creating Dynamic Drawings on an Isometric Dot Grid

Primary Krypto	**Proof Without Words: Pythagorean Theorem**	**Random Drawing Tool - Individual Trials**
 Playing a Game with Five Number Cards and Arithmetic Operations to Create a "Target" Number	 Proving the Pythagorean Theorem "Without Words" Using Geometry	 Exploring the Relationship Between Theoretical and Experimental Probabilities
Product Game	**Scale Factor**	**Shape Cutter**
 Exercising Factor and Multiples Skills	 Exploring the Relationship Between Two Shapes as the Scale Factor Changes	 Drawing, Cutting, Sliding, Turning, and Flipping Shapes
Shape Sorter	**Shape Tool**	**The Quotient Café**
 Sorting Shapes According to Their Properties Using Venn Diagrams	 Drawing, Coloring, Pasting, Slicing, Rotating, Reflecting, Expanding, and Contracting Various Shapes	 Dividing Using the Method of Partial Quotients (aka, "Chunking")
Tower of Hanoi	**Vector Investigation: Dual Vector, Airplane Storm Chaser**	**Volt Meter**
 Solving a Puzzle by Moving Disks from One Peg to Another	 Adjusting the Magnitude and Direction of Vectors to "Fly" a Plane	 Using the Voltage of Batteries to Explore Positive and Negative Numbers

The following is an example of description about one of the applets, called **Dynamic Paper**. It explains the use and instructions how to use the applet.

Dynamic Paper

Need a pentagonal pyramid that's six inches tall? Or a number line that goes from -18 to 32 by 5's? Or a set of pattern blocks where all shapes have one-inch sides? You can create all those things and more with the Dynamic Paper tool. Place the images you want, then export it as a PDF activity sheet for your students or as a JPEG image for use in other applications or on the web.

Instructions

This applet allows you to create the following:

- **Nets** – two-dimensional outlines of three-dimensional shapes, including regular polyhedra, prisms, pyramids, cylinders and cones
- **Graph Paper** – coordinate graphs, polar coordinates, logarithmic graph paper
- **Number Lines** – including positive and negative coordinates
- **Number Grids** – hundreds boards and the like
- **Tessellations** – tiling patterns involving triangles, quadrilaterals, and hexagons
- **Shapes** – pattern blocks, attribute blocks, and color tiles
- **Spinners** – up to 16 sectors, with adjustable sizes

Select the tab for the type of image you want, enter values for the parameters, and click **Add** to insert an image into the workspace. On the workspace, you can adjust the size of the image; to change other characteristics, you will need to adjust the values of the parameters on the dashboard.

The **Reset** button will restore the default values for all parameters.

The **+** and **–** keys can be used to create additional pages, and the arrow keys can be used to move back and forth between them. Images that you create will be inserted on the current page; if you delete that page, the images will be deleted, too.

The eraser button will delete all images on the current page. The trash can icon will delete all pages and erase all image, and a new blank document will be started.

*More information is available under the **Help** tab in the applet.*

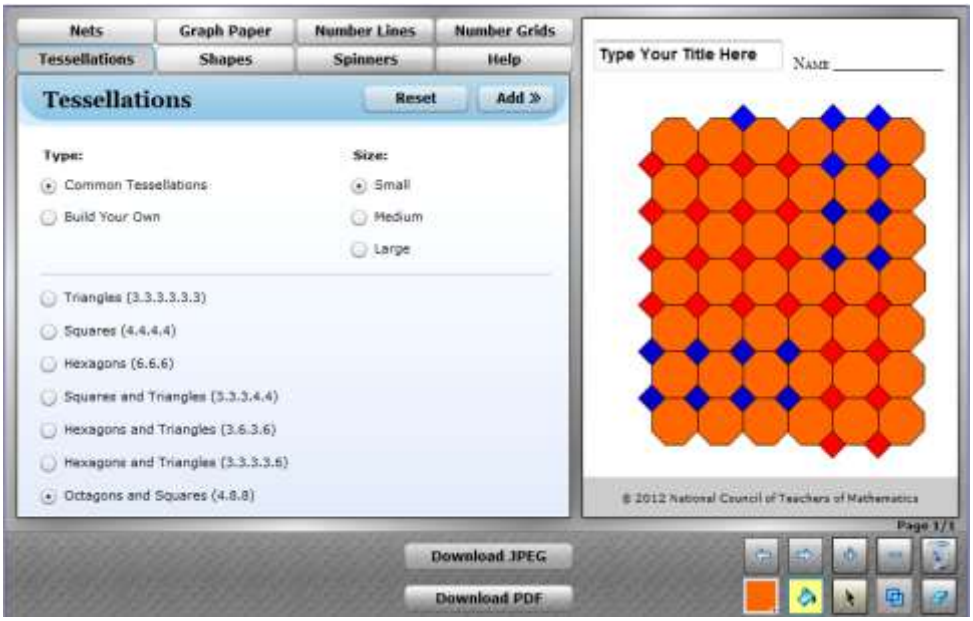


Figure 1 Dynamic Paper from Illumination of NCTM

Figure 1 Dynamic Paper from Illumination of NCTM shows the display of **Dynamic Paper**. It consists of two windows: the left window contains menu buttons, the title of selected menu, configuration

buttons for each menu, **Reset** and **Add>>** buttons; and the right window is the drawing pad used to draw appropriate picture. Under the two windows there two buttons, one to export the picture into **JPEG** file, the other to export the picture into **PDF** file. On the right button corner there is a collection of buttons to manipulate the picture. User can put the title of drawing and his/her name.

The tool can be used to create seven different mathematical pictures: (1) nets of 3D figures, (2) different types of tessellations, (3) different types of graph paper, (4) different types of 2D geometrical figure, (5) number lines, (6) spinners, and (7) number grids. It can also create multiple page pictures. The details of how to use the tool can be read through the menu **Help**.

All tools from Illumination of NCTM are very useful for teaching and learning mathematics. They can be used to create mathematics problem, to solve mathematics problem, and to explore mathematics concepts and properties. They are freely accessible for all teachers and students as long as they have Internet connection.

2. WisWeb by **Freudenthal Institute** (http://www.fi.uu.nl/wisweb/en/)

The Web site **WisWeb** by Freudenthal Institute (FI) of the Utrecht University, Netherlands, is a Web site for secondary mathematics education. It has **DME (Digital Math Environment)**, which is a simple web-based learning environment providing instructional modules in the forms of Java applets. One part of the DME is **DME learning trajectories**, which consist of two trajectories: (1) Trajectory 1 (Negative Numbers) and (2) Trajectory 2 (Geometry 3D, not fully translated yet). On the DME, students can create their own account. This account allows students to log in, and then save their work with applets. Using an account, students can leave their work and log back in to continue their work any time and any place they have Internet connection.

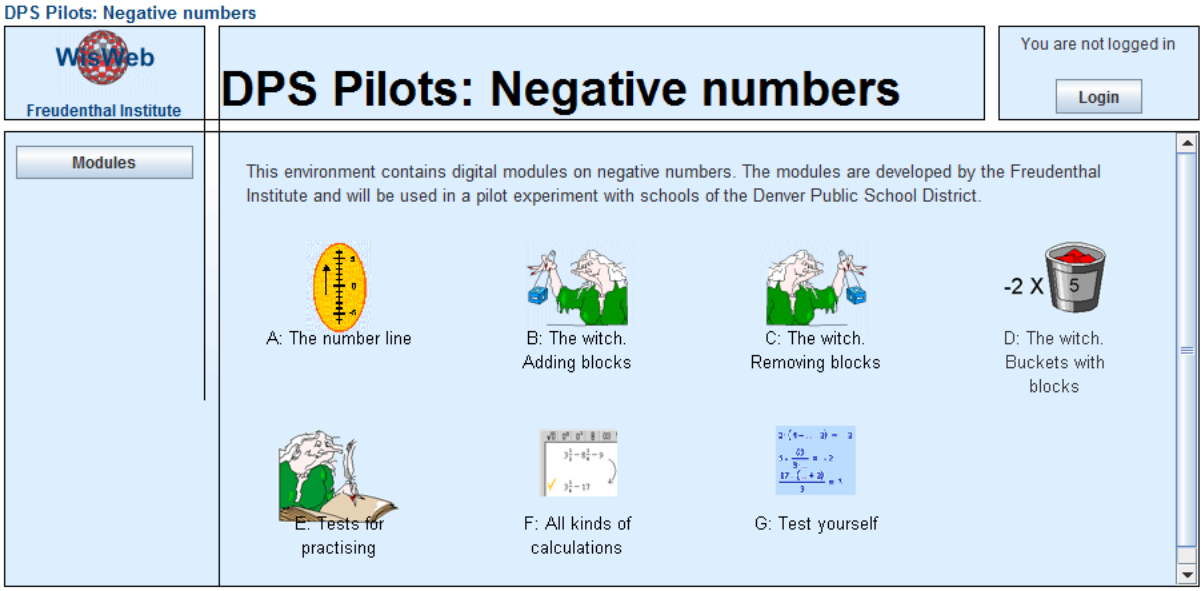


Figure 2 The DME Learning Trajectory 1 of Freudenthal Institute on Negative Numbers

Figure 2 The DME Learning Trajectory 1 of Freudenthal Institute on Negative Numbersshows the DME Learning Trajectory 1 (Negative Numbers). It has seven modules: the number line, the witch adding blocks, the witch removing blocks, the witch buckets with blocks, test for practicing, all kinds of calculations, and test yourself. Each module consists of a collection of interactive problems, where students can fill answers and get scores. **Figure 3** The DME Learning Trajectory 2 of Freudenthal

Institute on Geometry 3Dthe DME Learning Trajectory 2 on Geometry 3D. It has six modules: looking from all sides, views, building with blocks, drawing in space, using planes in objects, and making nets out of objects. Some modules are similar to those available on the NCTM's Illumination. Unfortunately, most of the Geometry 3D DME's modules are still in Dutch.

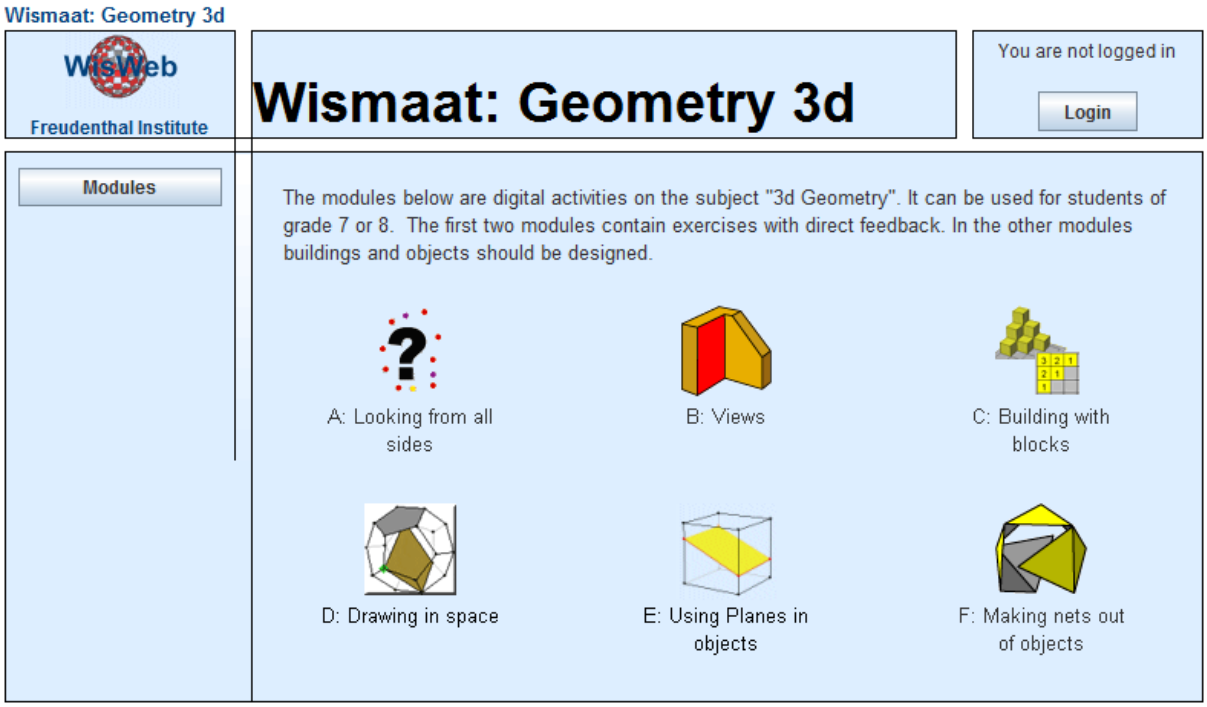


Figure 3 The DME Learning Trajectory 2 of Freudenthal Institute on Geometry 3D

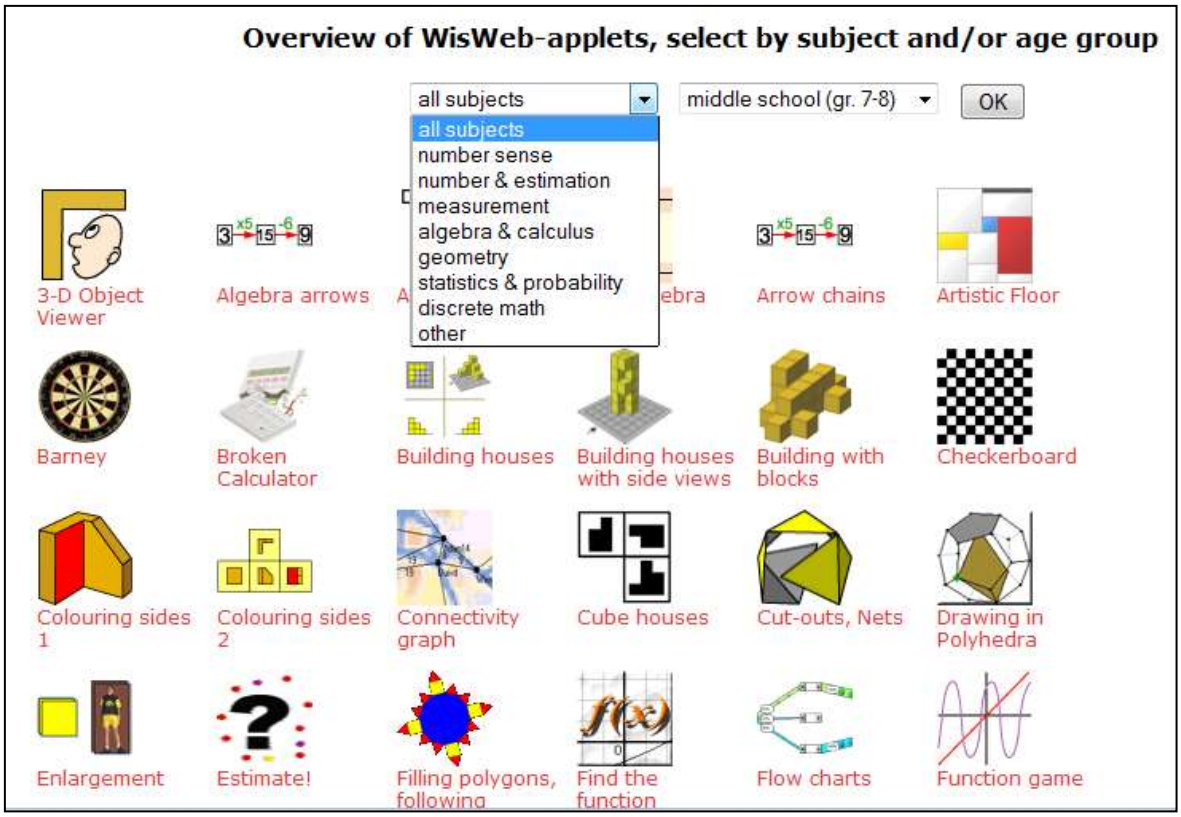


Figure 4 A Collection of Java Applets from FI's WishWeb

The **WisWeb** also provides a collection of a lot of Java applets for doing mathematics activities for different grades of students, including grade 7 - 9. **Figure 4** A Collection of Java Applets from FI's WishWebshows a collection of Java applets from FI's WishWeb. Students can select the desired topics for their grade. There are also special selections of applets: (1) applets with algebra courseware on linearity (**Figure 5**), and (2) applets with mathematics in context (**Figure 7**). An example of worked Java applet on solving linear equation with balance-strategy is shown on **Figure 6**.

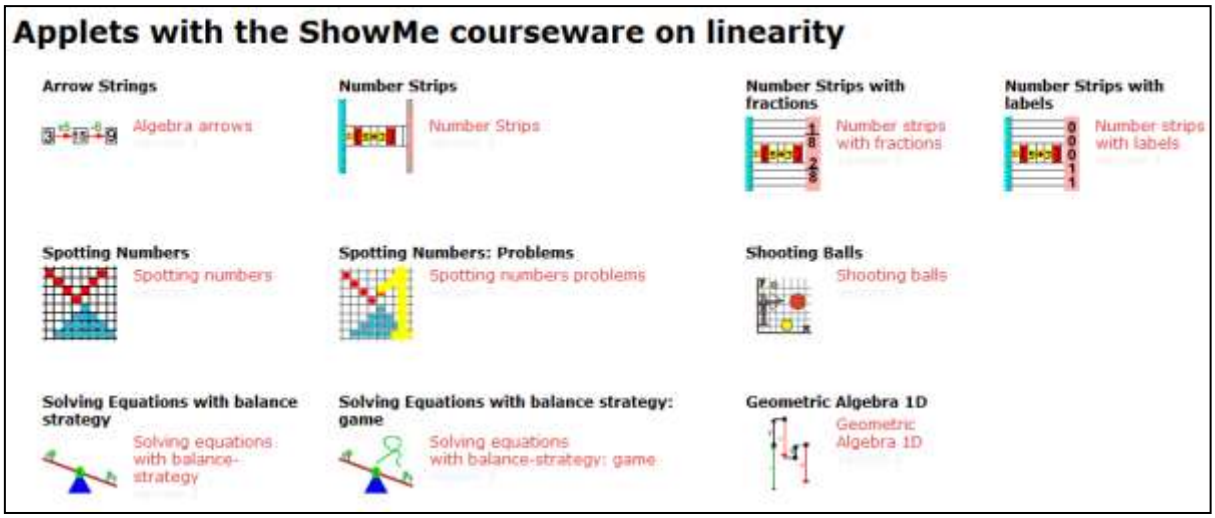


Figure 5 Collection of Java Applets with the ShowMe courseware on linearity

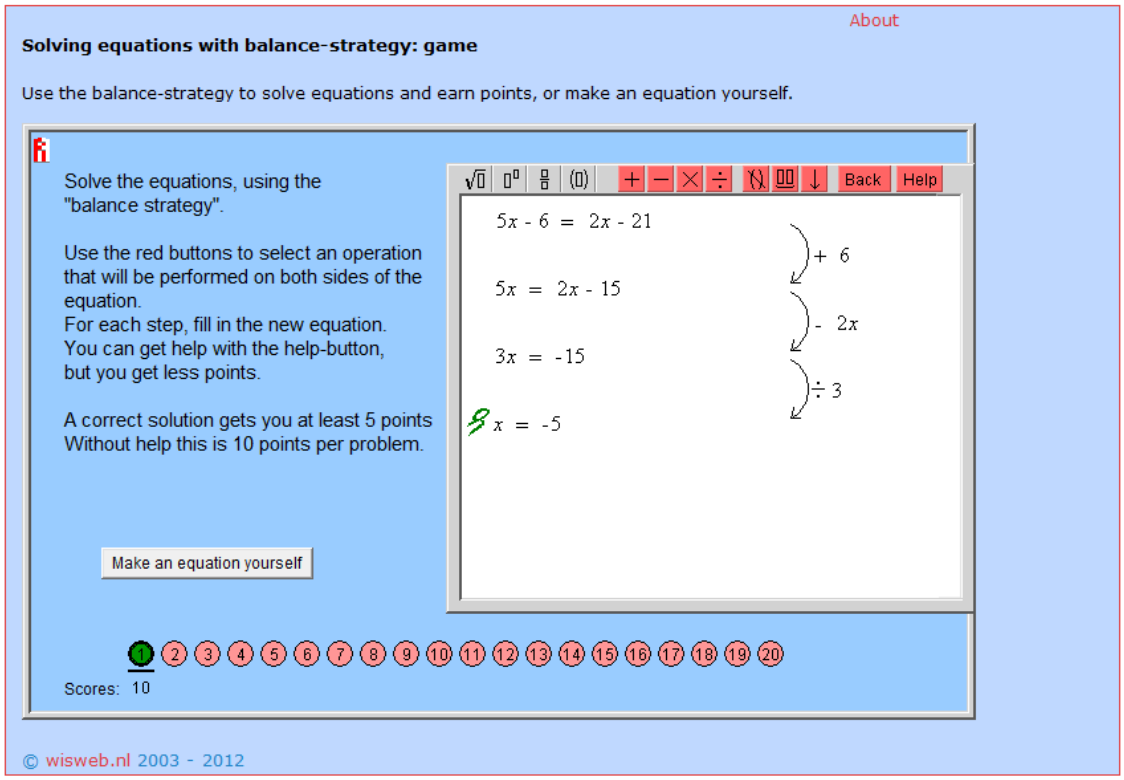


Figure 6 An example of worked Java applet on solving linear equation with balance-strategy

Both the Web sites **WisWeb** and the NCTM's Illumination are very good learning resources for secondary school teachers and students. Their various interactive learning modules in the forms of Flash or Java applet allows students to model contextual problems, create mathematical problems, explore mathematical properties, practice mathematical skills, and solve problems interactively.

Therefore, they can be used to support RME-based teaching and learning mathematics at Junior High Schools. Even some applets are designed in the form of interesting and challenging mathematical games. This makes students enjoy and feel exciting during their learning process. This is something that is a benefit for mathematics education.

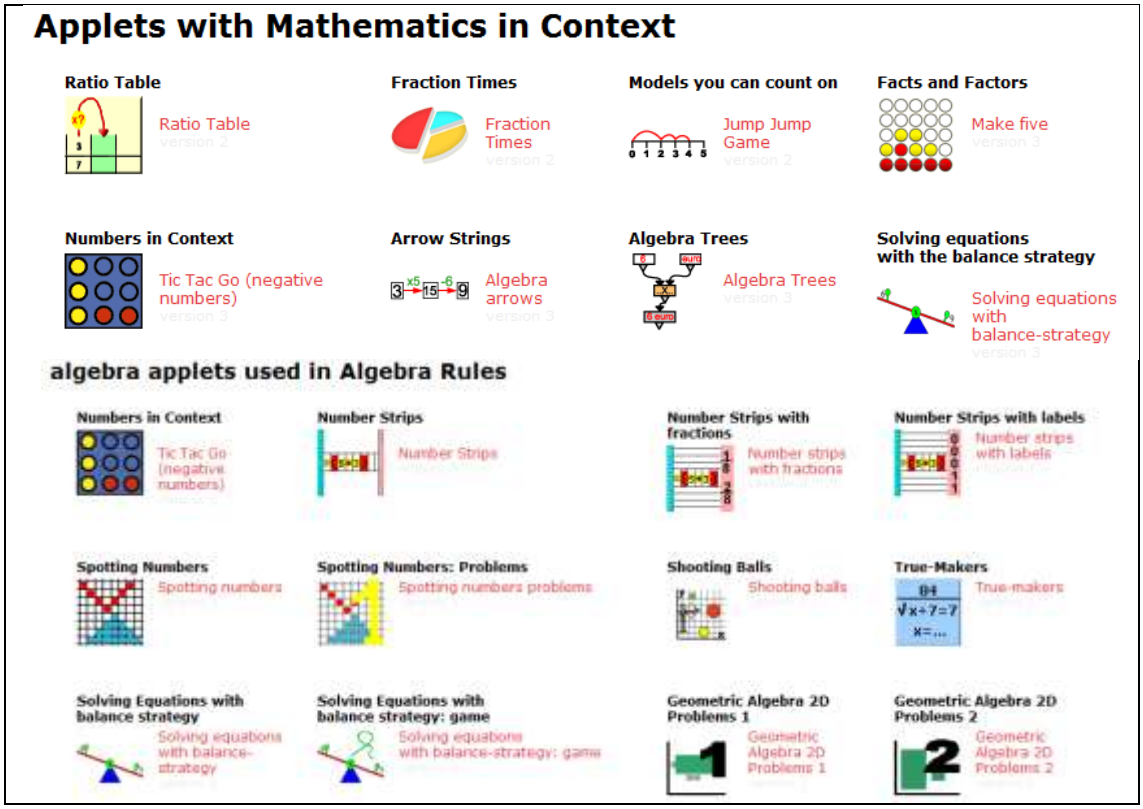


Figure 7 A Collection of Java applets with mathematics in context

In addition to professional organizations and research institutions which developing and providing web-based interactive mathematics learning modules, there are also software developers, either the commercial or non-commercial developers, who provide learning resources to support the use of their educational software. Among them are **Geometer's Sketchpad** (**Key Curriculum Press**) and **GeoGebra**.

3. Geometer's Sketchpad Resources by Key Curriculum (http://www.keycurriculum.com)

Geometer's Sketchpad is one of dynamic mathematics software developed by **Key Curriculum**, one of the leading experienced mathematics education software developers in the world, who has mission to (http://www.keycurriculum.com):

- **engage students with effective, relevant, high-quality mathematics software and instructional materials** that open their eyes to math in the world around them, develop both conceptual understanding and skills, and ignite their interest in learning;
- **support mathematics educators** by partnering with them to promote an inclusive and compelling learning environment that facilitates their success in meeting the educational needs of all students;
- **advocate for research, ideas, strategies, and policies that lead to excellence and equity** in education, as well as a better educational experience for all students; and

- **provide a respectful, collaborative, and forward-thinking workplace** that promotes open communication, values people, nurtures their ideas, and helps Key Curriculum achieve sustainable growth.

The **Geometer's Sketchpad®** is one of the world's leading software for teaching and learning mathematics and also one of the most widely used and effective technologies for school mathematics. It is paid software, but everyone can download and try freely for one month, and price for individual is relatively cheap ($69.95). **Sketchpad®** can be used by students at all levels, from third grade through college. It provides a tangible, visual way to learn mathematics that increases students' engagement, understanding, and achievement, and also make mathematics more meaningful and memorable. As mentioned by the Web site Key Curriculum,

> "Middle school students can build their readiness for algebra by exploring ratio and proportion, rate of change, and functional relationships through numeric, tabular, and graphical representations.
>
> **Sketchpad** is the optimal tool for interactive whiteboards. Teachers can use it daily to illustrate and illuminate mathematical ideas. Classroom-tested activities are accompanied by presentation sketches and detailed teacher notes, which provide suggestions for use by teachers as a demonstration tool or for use by students in a computer lab or on laptops."

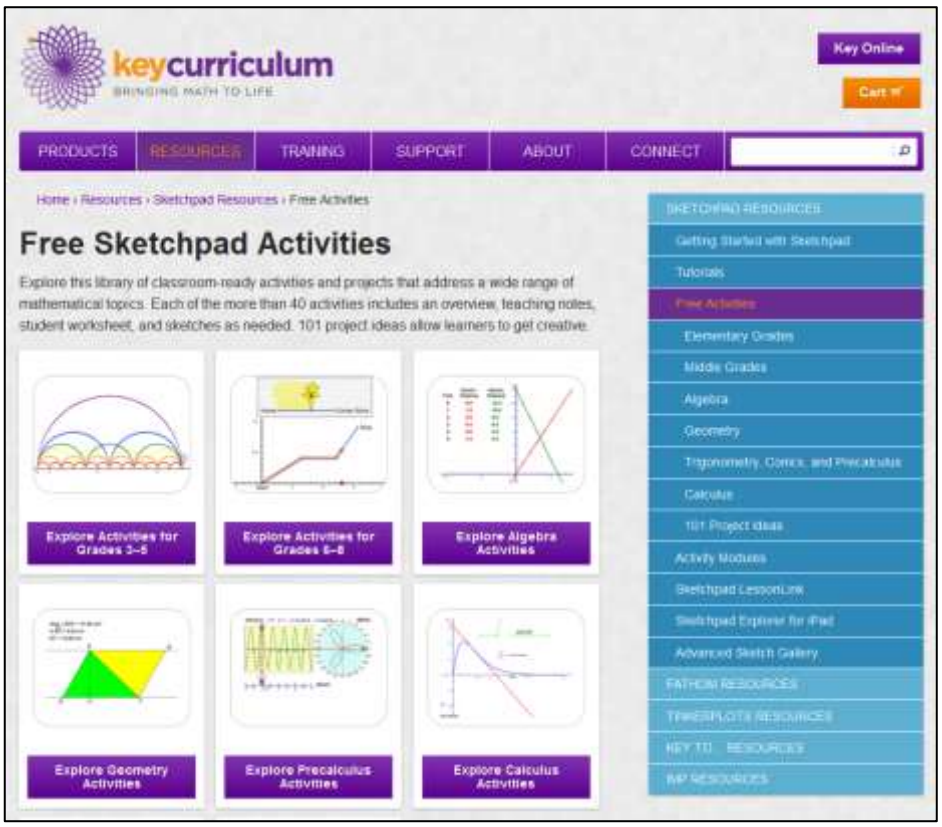


Figure 8 The Screen of Sketchpad Activities

The Web site provides very richful learning resources for teaching and learning mathematics using the **Geometer's Sketchpad**. There are **Free Sketchpad Activities Library**, **Sketchpad Sketch Exchange**, and **Sketchpad Lesson Link**. The Free Sketchpad Activities Library is a collection of freely downloadable classroom-ready activity modules for a wide range of mathematics topics for all students including middle school (Junior High School) students. This library is developed by Key

Curriculum. Each module includes an overview, teaching notes, student worksheet, and Sketchpad files that can be opened on Geometer's Sketchpad by which students can make interactive activities and exploration on the topic. **Figure 8** shows the screen of Sketchpad Activities. The activities are grouped based on the grade and also based on the topic, i.e. there activities for grades 3 – 5, grades 6 – 8, activities for Algebra, Geometry, Pre-calculus, and Calculus. One module for Junior High School (grade 6 – 8) is **Quadrilateral Pretenders: Classifying Quadrilaterals**. Using this module, students can drag the edges and vertices of various quadrilaterals to discover which are constructed to have specific characteristics. As they make distinctions on the basis of these characteristics, they deepen their understanding of the definitions of various quadrilaterals, their properties, and the relationships among them (see **Figure 9**).

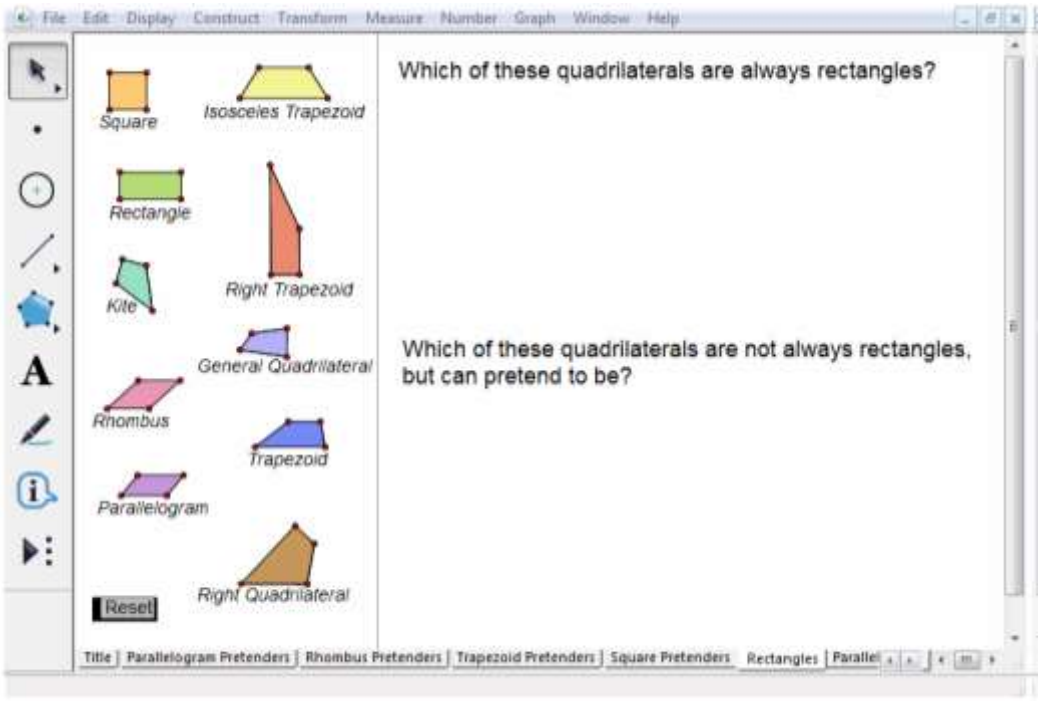


Figure 9 Sketchpad Activities on Quadrilateral Classifications

The Sketchpad's Web site also has **Sketchpad Sketch Exchange**, an online community for sharing Sketchpad activities, tips, questions, and ideas (see **Figure 10**). At Sketch Exchange registered (free) teachers and students can do:

- **upload** sketches and activities,
- **post** links to their own Web site or videos,
- **participate** in a community forum,
- **browse** the content on the site by grade level, topic, or tags,
- **download** sketches and activities for use in their classroom, and
- **collect** their favorite activities and access the sketches directly from Sketchpad.

The Sketch Exchange provides featured activities, recent Forum posts, and the Tip of the Week. It has the **Library** tab (to find activities by content area, grade level, or tags) and the **Community** tab (**Forum** discussions). To rate an activity, save activities as favorites, post a comment or question on the Forum, or post materials, a teacher or a student needs to have a user account, which is freely can be signed on at keyonline.keypress.com. Logged in user will have **My Files** and **My Favorites** tabs to save his/her own files and favorite activities.

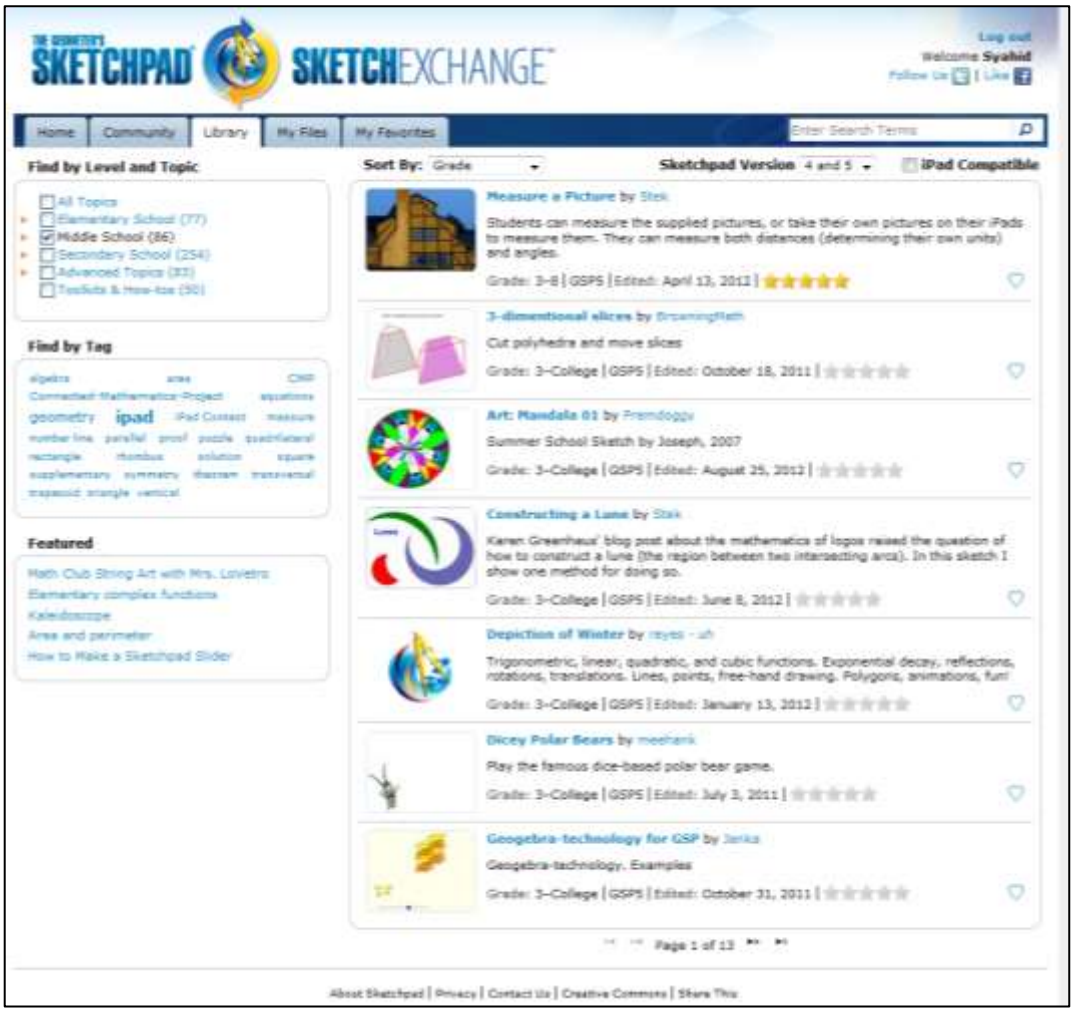


Figure 10 The Screen of Sketchpad Sketch Exchange

Figure 11 The Screen of Sketchpad LessonLink

The other free service provided by Sketchpad's Web site is **Sketchpad LessonLink®** (see **Figure 11**). It provides curriculum for The Geometer's Sketchpad® and enhances mathematics lessons with more than 500 searchable dynamic activities and demonstrations aligned to textbooks, state standards, and the Common Core State Standards for grades 3–12. Activities include teaching notes, student worksheets, pre-built sketches, and links to tip sheets and videos that make it easy for teachers and students to learn and review Sketchpad skills. Activities can be searched by key word, grade level, Sketchpad skill level, activity duration, or classroom setting (whole class, pairs, or independent practice). Teachers can organize activities into folders to share with their colleagues or publish for their students to use in the lab or at home.

There are also free software developers who provide resources for teaching and learning mathematics. One of them is GeoGebra.

4. GeoGebra (www.geogebra.org)

GeoGebra is free and multi-platform Java-based dynamic mathematics software for all levels of education that joins geometry, algebra, tables, graphing, statistics and calculus in one easy-to-use package. It was created by **Markus Hohenwarter** (markus@geogebra.org) from Johannes Kepler University Linz, Austria. GeoGebra is one of the very rapid developing dynamic mathematics software that has been used at schools and universities with millions of users around the world. The developer Team is directed by its creator and supported by experts from many universities in Europe as well contributors from around the world. The software has unique features such as:

- graphics, algebra and tables are connected and fully dynamic;
- easy-to-use interface, yet many powerful features;
- authoring tool to create interactive learning materials as web pages;
- available in many languages for our around the world; and
- free and open source software.

As free educational software, GeoGebra can be downloaded and freely used by teachers and students for teaching and learning mathematics at schools, including Junior High Schools. In the GeoGebra Web site, there are some useful resources for teacher and students. There is **GeoGebraTube** (http://www.geogebratube.org/), the official repository of GeoGebra constructions and GeoGebra related resources (see **Figure 12**).

On **GeoGebraTube** users can explore GeoGebra materials (worksheets) contributed by GeoGebra developers and users. Unfortunately, the materials are not classified based on mathematical topics neither on grade levels. They are grouped according to **Featured Materials, Newest Materials, Best Worksheets**. The materials types are classified into **worksheet, tool, collection, lesson, publication, tutorial,** and **miscellaneous**. There are also some popular tags to help users to find materials on certain mathematical topics. Registered and logged in users can upload, share, and create collection on GeoGebraTube.

Figure 13 shows an applet created using GeoGebra and available from GeoGebraTube. This applet can be used in teaching and learning topic on Area of Circle for Junior High School. Using the applet, student can find the formula for the area of circle, by dissecting the circle into a number of parts, then rearranging the parts into parallelogram shape and notifying the dimensions of the shape.

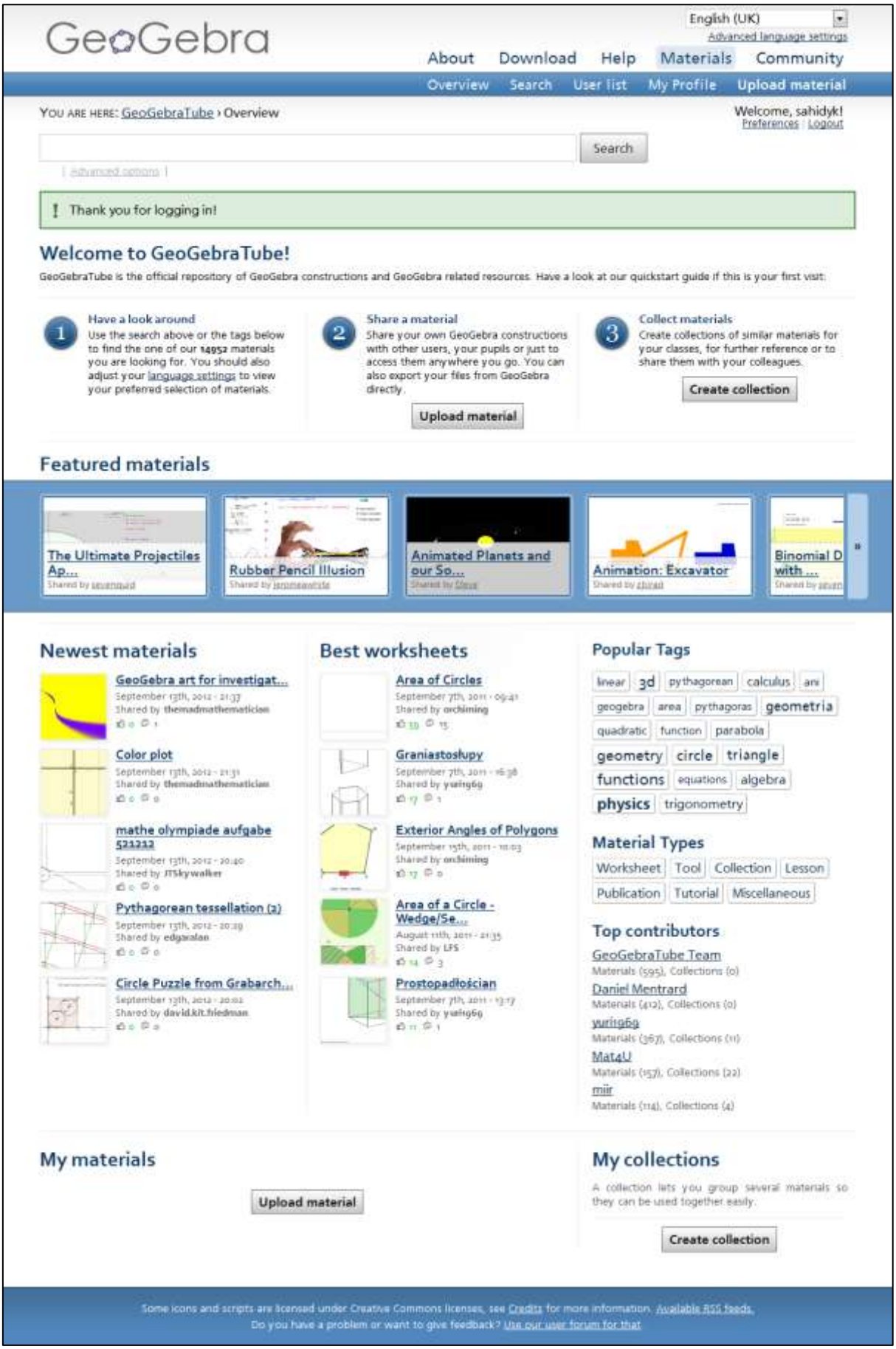


Figure 12 The Home Page of GeoGebraTube Containing GeoGebra's Resources

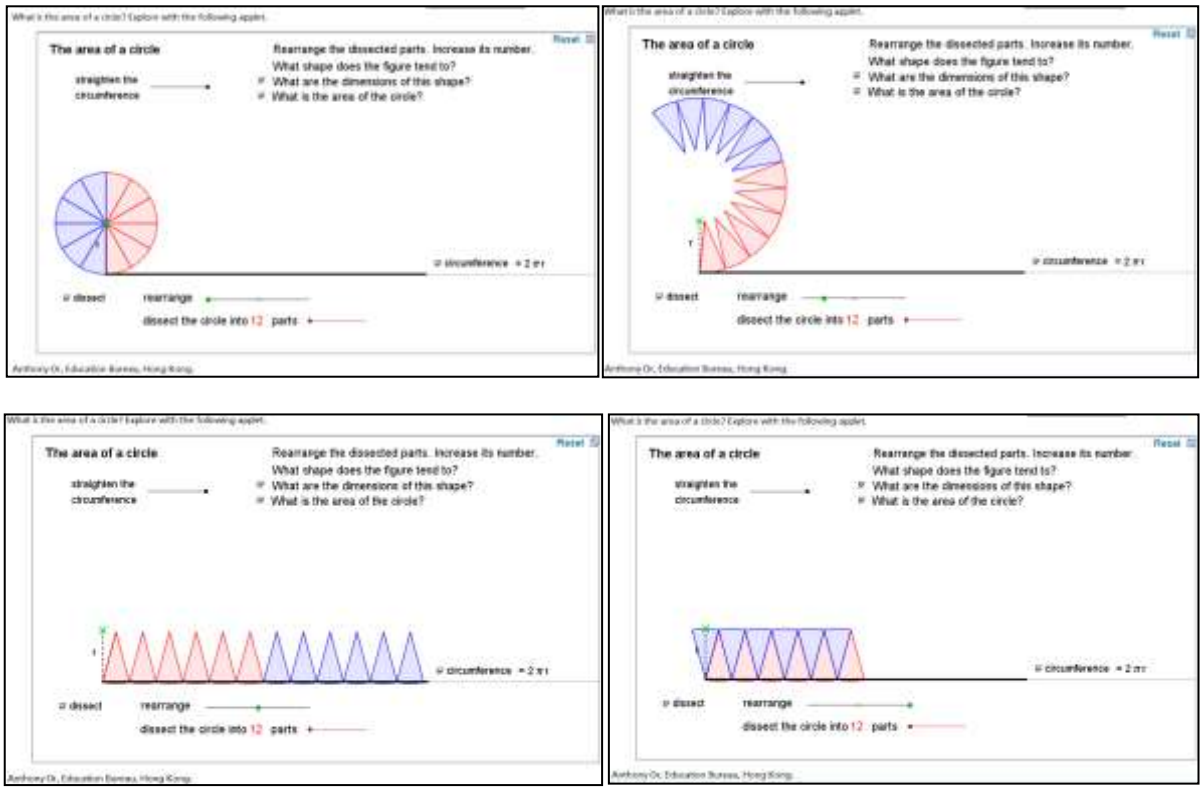


Figure 13 Finding the Area of Circle Using GeoGebra's Applet

There are many other resources on the Internet that can be used by teachers in their teaching and learning processes. The easiest way is to ask **Google** by using appropriate key words, such as "mathematics activity", "mathematics students' activity", "learning mathematics middle school", etc. The following are a few of selected online resources on mathematics education for teachers and students.

NRICH enriching mathematics - http://nrich.maths.org/public/

The NRICH Project aims to enrich the mathematical experiences of all learners, to provide professional development for teachers wishing to embed rich mathematical tasks into everyday classroom practice. NRICH is part of the family of activities in the Millennium Mathematics Project by University of Cambridge (1997 – 2012). The Web site contains thousands of free mathematics enrichment materials (problems, articles and games) for teachers and students from lower primary school to upper secondary school. All the resources are designed to develop subject knowledge, problem-solving and mathematical thinking skills. The Web site is updated with new material on the first day of every month.

Mathematical Interactivities - http://mathematics.hellam.net/

The **Mathematical Interactivities** Web site contains puzzles, games and other online educational resources created and organized by David Hellam. The Web site provides popular math games online, such as: the **Frog Puzzle** (move the frogs across the pond by hopping or sliding and work out how many moves would need to shift any number of frogs), Flash Algebra (interactive tools to solve linear, quadratic and simultaneous equations), Mathematical Matches (a series of flashcard/concentration puzzles - sometimes with an added interactive twist, which covers a wide range of mathematical topics), Interactive 100 Square Chart (patterns made by multiples of numbers in a 100 square), and Tessellations (Escher-style tessellations and Islamic patterns in Logo and Flash). Other games and puzzles are grouped

into **number puzzles, interactive algebra online, measurement, shape and space – interactive geometry, fun with handling data,** and **puzzles – maths investigation – trivia.** The Web site also provides links to other Web sites created by the same author.

Math Cats - http://www.mathcats.com

The **Math Cats** Web site (2000 - 2012) is devoted to math activities, crafts, and games developed by Wendy Petti and other teachers for elementary and middle school students for helping them discover the logic and beauty of mathematics. The Web Site provides six windows for doing different activities: 1) **Math Cats Explore** (more than 30 interactive puzzles and math-related activities that challenge students' creativity and understanding, such as **Crossing the River** problem, **OBBL Architecture Blocks, Tessellation Town on Tile Island, Old Egyptian Math Cats Fractions,** etc.), 2) **Math Cats love MicroWorlds!** (more than two dozen interactive MicroWorlds-based mathematics applets, such as **Broken Calculator,** which is a game to construct different calculations using a calculator with some keys are broken to produce a given answer), 3) **Math Crafts** (a collection of instructions to make offline craft projects based on mathematical and geometric principles, such as creating a number city or a polygon airport out of construction paper), 4) **Math Cats' Art Gallery** (a collection of math drawings created using **Logo**), 5) **Math Cats Attic** (collections of fascinating mathematical facts such as math terminology, math history, math in sports, and math in the human body), and 6) **Thinking Games** (a collection of mathematical games/puzzles such as **Guess My Number, 3D Tic Tac Toe,** arrange numbers in **Triangles** or **Hexagons**). The Web site has received many awards, honors, and reviews from several math organization and other educational institutions.

Waldomaths - http://waldomaths.com/index.jsp

Waldomaths (2001 – 2012) is a Web site created by a retired secondary mathematics teacher, Ron Barrow from West Kent area of England, United Kingdom, who is now living in a ruined farmhouse in France. According to the author, since the beginning **Waldomaths** has been used in more than 150 countries by hundreds of thousands of teachers and students of mathematics. The Web site provides resources for teachers and students of age 11 to 19 (lower secondary to upper secondary school). Teachers can download woksheets, presentations of some mathematics topics in Power Point and video formats, and some mathematical modules. Students can play interactive mathematical games, such as hidden mathematical words, and they can also do mathematics activities interactively using tens of Java applets for many topics in Junior and Senior High Schools.

There many other Web sites on the Internet that provides good resources for teaching and doing mathematics activities. However, teachers must be able to select the best Internet resources, such as the above examples, for their students and themselves.

In addition to using online resources from the Internet for supporting mathematics teaching and learning, teachers can also use offline mathematics software to create students activities. There are a lot of mathematics software available, either the commercial or the non-commercial (free) software, such as AutoGraph, Cabri 2D, CaR, GeoGebra, Geometer's Sketchpad, Grapes, MATLAB, MAPLE, MS Excel, etc. The next two sections will describe how to use MS Excel and Dynamic Mathematics software for creating and doing mathematics activities for Junior High School students.

C. Using Electronic Spreadsheets (MS Excel)

Electronic spreadsheets can be used in teaching and learning mathematics at Junior High Schools to explore a variety of mathematical concepts and to help students use numerical and graphical methods to solve problems (Abramovich, 1995; Abramovich & Nabors, 1998; Clements & Samara, 1997; Dugdale, 1998; Neuwirth, 1995, quoted by Hollylymre Stolrl Drier, 2001: 170). Furthermore, Drier explain,

> "These the uses of spreadsheets allow students to explore alternative solution processes that go beyond symbolic manipulation and can provide students with a deeper understanding of concepts embedded in a problem. One unique use of spreadsheets is the ability to interactively model and simulate mathematical situations. In much the same way scientists use a laboratory to discover and test scientific laws, mathematics teachers can use spreadsheets to create dynamic experiential environments for discovering mathematical relationships. … Interactive spreadsheets can be designed by a teacher to provide students with a laboratory-like environment to investigate a mathematical problem. An interactive spreadsheet can be designed such that the user can perform an action that changes the status of cells, formulas, or graphs in the spreadsheet. When the cells, formulas and graphs are linked, users can observe dynamic changes in multiple representations. … Such activities can facilitate students' engagement with mathematical concepts and their conceptualization of relationships among numerical, graphical, and algebraic representations. … "

Electronic preadsheet, such as *Microsoft Excel*, provides teachers and students with a creative tool for the study and teaching of mathematics, mathematical modeling, and mathematical visualization. It enables students to gain mathematical insights into a diverse range of interesting and significant applications in an engaging setting while they simultaneously acquire practical skills in using the principal mathematical tool of the workplace (Deane Arganbright, 2007).

The following examples are some limited number of teaching and interactive learning activities using MS Excel 2007. If teachers have limited skills in using MS Excel to develop interactive mathematics, they can find many ready for use mathematics worksheets using MS Excel from the Internet. Ask for some searching engine like **Google** to find the worksheets using appropriate key words, like "mathematics activities", "interactive workbook in Excel for mathematics", etc.

1. Fractions, Ratios, Decimals and Percentages (http://www.maths-it.org.uk/Flexcel/Flexcel.php)

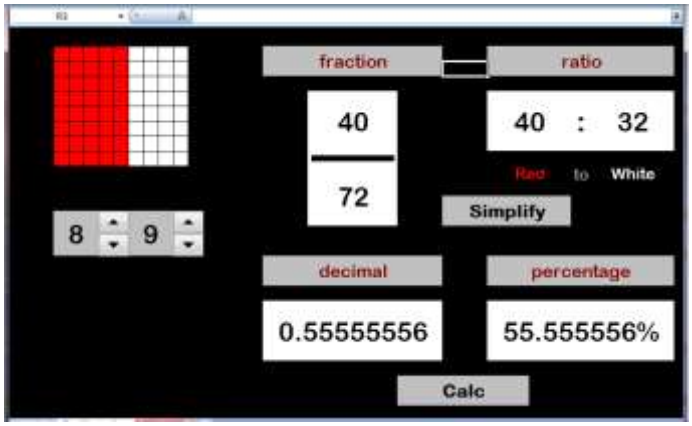


Figure 14. Interactive Fraction, Ratios, Decimals and Percentages using MS Excel

The **Fractions, Ratios, Decimals and Percentages** workbook (**Figure 14**) is aimed at improving students understanding of the equivalence of these values. It has several interactive features.

a. **Sizing the grid**. The size of the fractional grid can be made changed from 1x1 to 10x10 by clicking on the spinners to the right of the sizes under the grid.
b. **Changing red to white squares and back**. To change the color of the fraction grids from red to white and vice versa, just click on each individual square in the grid. Each color change will result in to a new fraction.
c. **Hiding and showing the values**. To hide or to show each of the values, numerator, denominator, percentage, decimal and each side of the ratio, just click on the cell.
d. **Simplifying the fraction and ratio**. The fraction and ratio will be shown unsimplified according to the current numbers of red square grids and of white square grids. To simplify the fraction and ratio, just click on the **Simplify** button.

This interactive Excel worksheet can be used in mathematics classroom for Junior High School, by which students can do the following activities.

a. Write a fraction, ratio, and decimal values that related to red and white colored grids proportion.
b. **How many shaded**. Show the students the worksheet with no red colored grids and give them one value of a fraction, ratio, decimal, or percentage, e.g. 2/3, 3: 7, 0.35, or 30%. Ask students how many squares need to be shaded to make the value. Using a computer connected to a projector, get one student to demonstrate the answer on the worksheet. Ask students to do the same activities for different grid sizes and values.
c. **Equivalent fractions**. Start a worksheet with certain number of red-colored grids and show the associate fraction. Ask students change the number of red-colored grids to make different value, e.g. 1/2, 3: 7, 0.35, or 30%. Using a computer connected to a projector, get one student to demonstrate the answer on the worksheet. Ask students to do the same activities for different grid sizes and values. Ask questions such as "Can we shade in 1/3 of the grid? If not why not?"
d. **Investigate.** Ask students to show the fraction and ratio cells and to investigate the relationship between the fractions and ratios for various grid sizes and a different number of squares shaded. Tell them that they can use the **Simplify** button to simplify the fraction or calculator to calculate the fraction whenever they want. Tell them that they will have to write, present or discuss their findings.

2. Solving Price Problems

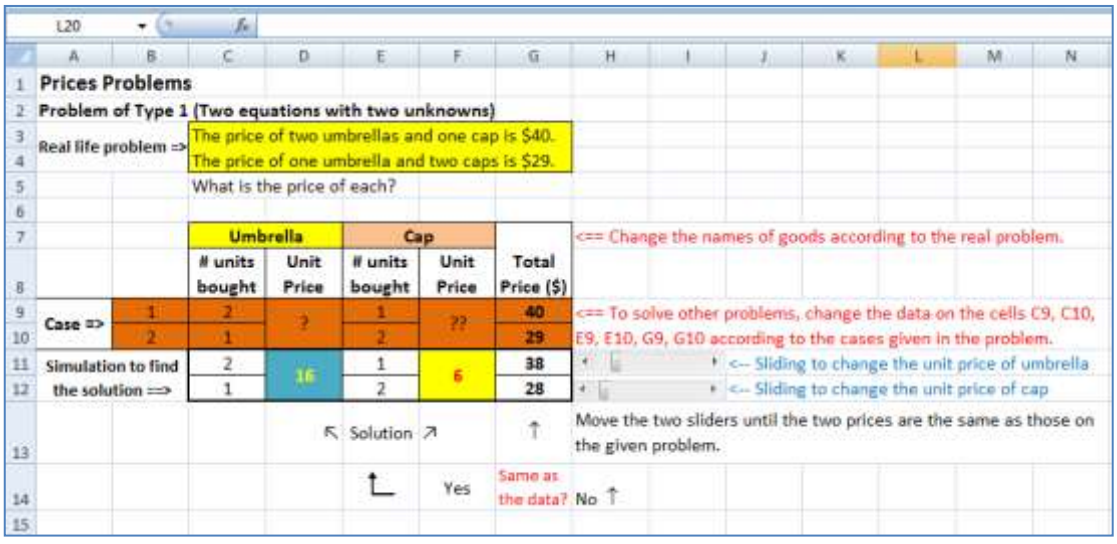


Figure 15 Interactive worksheet for designing and solving price problems

The worksheet shown on **Figure 15** is aimed to design and to solve price problems without using mathematical solution techniques. After the problem is set, students can fill the data into the appropriate cells and then interactively find the solution by moving the sliding bars to the right or to the right to obtain the right prices. This worksheet is very useful for teaching and learning system of linear equations with two variables (SLE2V), before introducing the solutions methods such graphical method or elimination and substitution method. This worksheet support exploration to find the solution of an SLE2V by trying different values of each variable. Instead of using the sliding bars, the students may enter the desired value in the unit price cells, but this is less practice, because they have to press ENTER to see the result.

Similar worksheet, but for determining the right unit prices if the total price is given is shown on the **Figure 16**. Using the worksheet students can interactively determine the right unit prices by moving the sliding bars until the total price is the same as in the problem. This kind of problem belongs to open ended problem, because probably there are more than one solution.

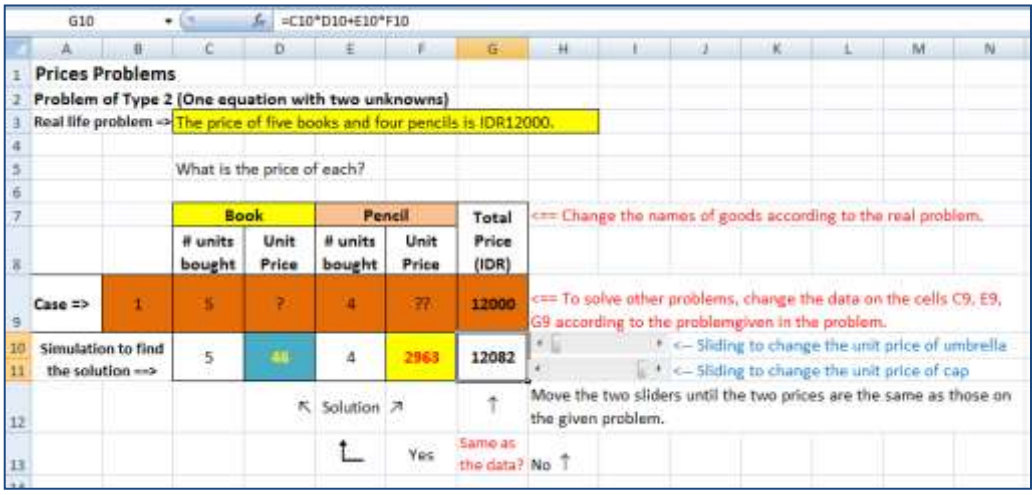


Figure 16 Interactive worksheet to find the right unit prices for a given total price

3. Exploring Line

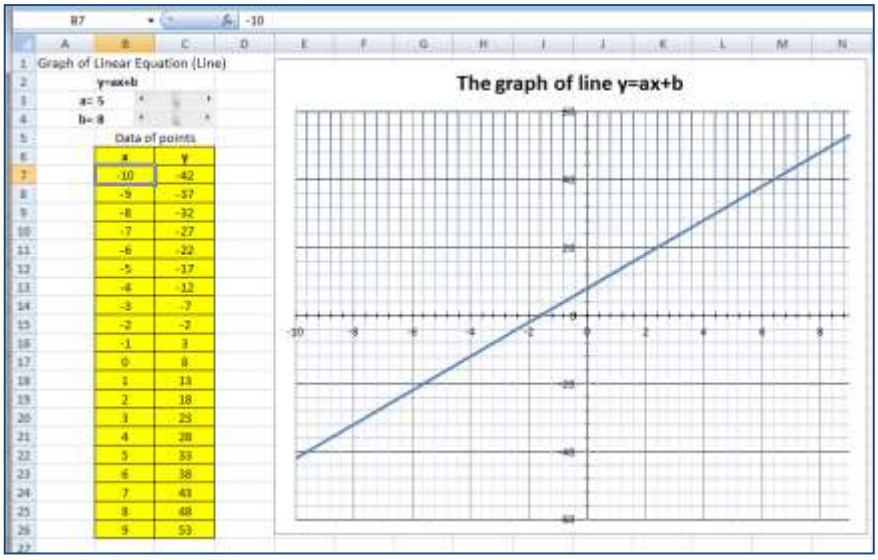


Figure 17 Investigating the properties of line $y = a.x + b$

The power of electronic spreadsheet like MS Excel to change values interactively using sliding bars (form control) is very useful to create interactive worksheets as shown in the previous examples. In this example, the worksheet also uses sliding bars to change the values of a and b. When students change the values, they will see the effects on the data as well the graph (line). Therefore, they can investigate the effect of change in a to the line, and the effect of b to the line.

4. Determining the Formula of a Number Pattern & Sequence Generator

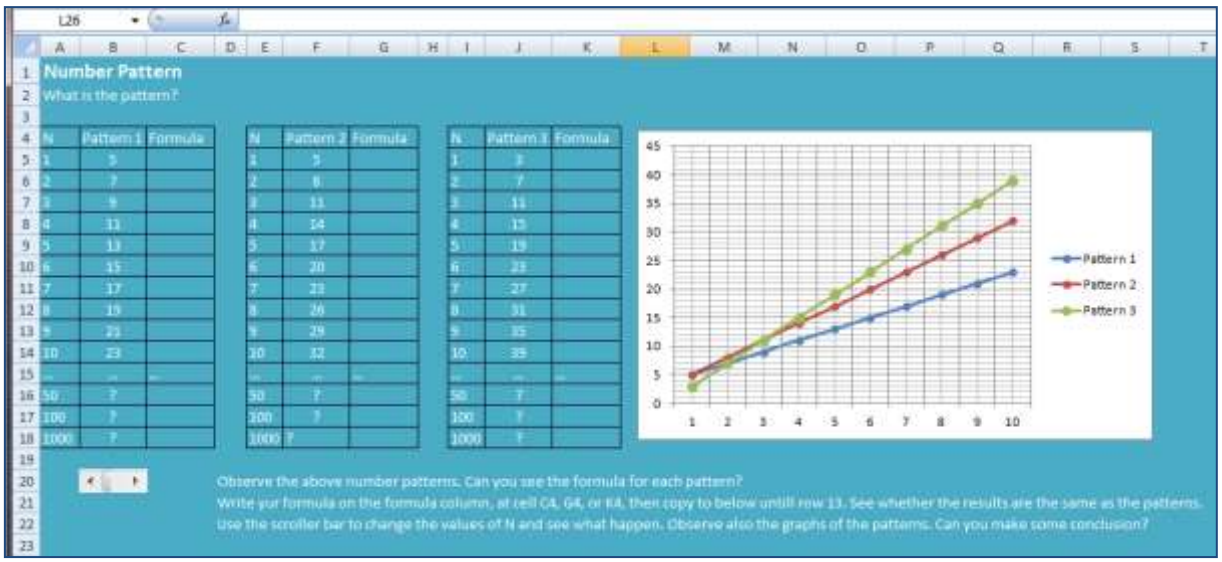


Figure 18 Finding the formula for a number pattern

In this worksheet, students are exposing with three different number patterns and they have to find the formula of each pattern. After they write a formula, they can copy to cells below the formula and see whether the result is the same as the given pattern. They can also change the value of N by using the sliding bar and observe what the effect to the pattern and also the graphs.

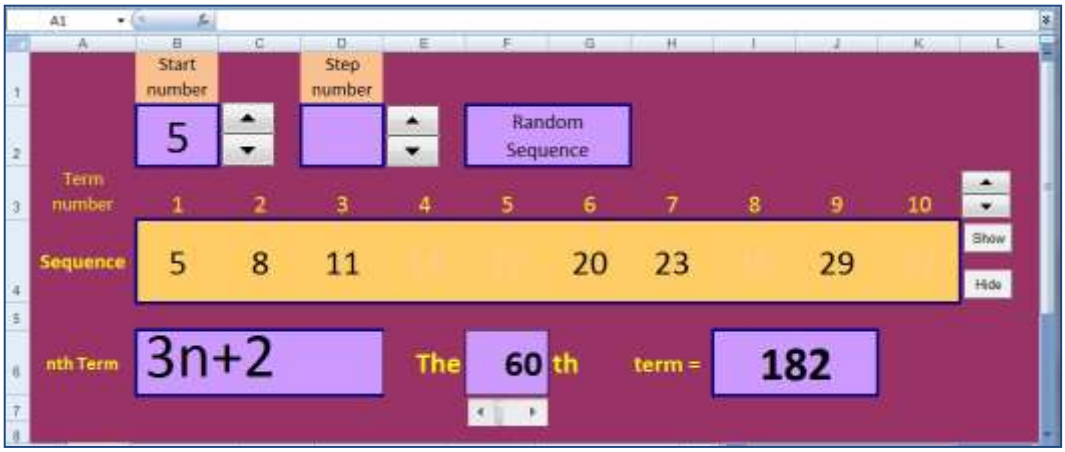


Figure 19 Interactive Arithmetic Sequence using MS Excel

Figure 19 shows another interactive worksheet on sequence. This worksheet can improve students understanding arithmetic sequences. Using the worksheet, students can do the following activities:

1. Displaying or hidding the values of start number, step number, the some terms (numbers) in the sequence, the general n-th term formula, the n-th term and its value, by clicking them or using

associated scroller bar (for start number, step number, the n-th term, and term numbers). The sequence can also be shown or hidden in entirety using the buttons (**Show**/**Hide**) at the end of the sequence.

2. Creating different arithmetic sequences using three ways: by using the spinners (scroller bar), by typing the values of star number and step number, and also, by clicking the Random Sequence button. Once they have done, they can do the same thing as in 1).
3. Guessing the value of certain terms in the sequence, when they have been hidden, or when the term numbers do not appear on the list, then check their asnwer, by using the spiner on the right of the term numbers or under the n-th term, or clicking the missing terms.
4. Determining the general formula the a sequence (when it has been hidden), then checking it on formula cell.

5. Target Number Game

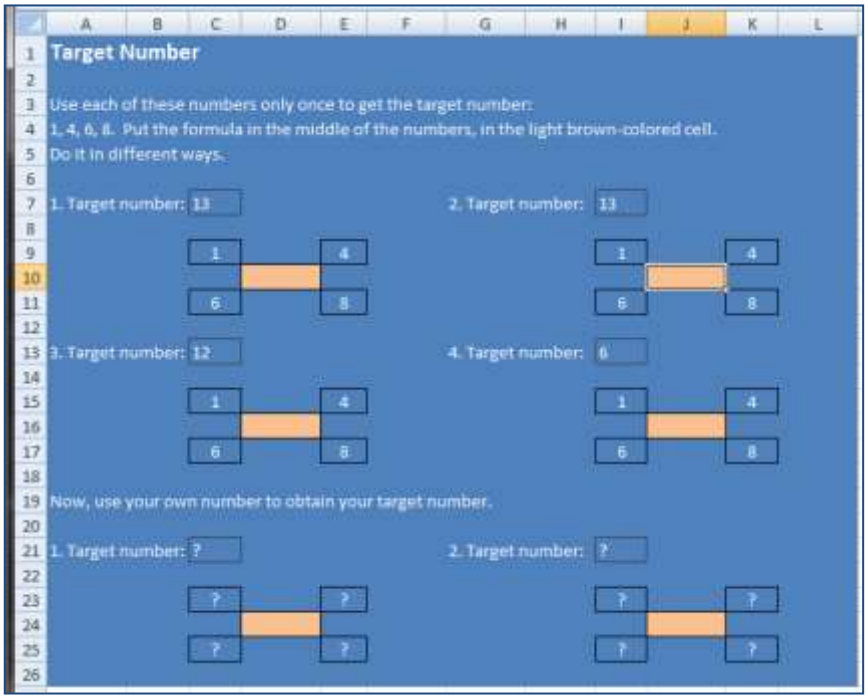


Figure 20 Constructing operations using each of four different numbers once to get the target number

Using MS Excel, teachers can also create interactive mathematics games like target number game as shown in the **Figure 20**. In the worksheet, students can explore different operations using four different integers, each once, to get the desired target number. They may write a formula, and check if the result is the same as the target number. If not, they have to revise the formula. If yes, they may write different formula giving the same result. They can also determine their own target number using their own four different integers.

There are other unlimited ways to use the powerful and accessible tool of electronic spreadsheet, like MS Excel, and its outstanding graphic features creatively in teaching a surprising number of mathematically oriented topics. MS Excel can also be used to create eye-catching animated graphic displays and integrate more fun into the study of mathematics. This can be done by using visual basic (VB) programming in MS Excel. Of course this requires advanced skills in MS Excel and VB

programming. However, again teachers do not be worry, because there are many ready to use interactive worksheets that can be found and downloaded from the Internet.

The next section will describe another ICT use, which is dynamic geometry software, in teaching and learning mathematics.

D. Using Dynamic Mathematics Software

As mentioned before, there are a lot of dynamic mathematics software. This section will only discuss on using dynamic geometry software in teaching and learning mathematics for Junior High Schools. Dynamic geometry software (DGS) or interactive geometry software (IGS) are computer programs which allow one to create and then manipulate geometric constructions (figures), in plane (2D) or solid (3D) geometry. In most DGS, one starts construction by putting a few points and using them to define new objects such as lines, triangles, quadrilaterals, circles or other points. After some construction is done, one can move the starting points and see how the construction changes (Wikipedia: Interactive geometry software, 30 August 2012). The main different between dynamic geometric figures and static geometric figures which are produced using non-dynamic geometry software such as MS Word or other drawing tools (which actually pure mimic paper and pencil) is object dependency. In a dynamic geometry software there are independent objects such as starting points, and dependent objects such as lines, circles, triangles, and quadrilaterals constructed from the starting points, which will change when the independent objects are moved, but the main geometrical property of the dependent objects will not change. In general drawing tools, all static figures are independent each other, meaning that changing one object will change the main property of other objects.

For example, using dynamic geometry software we first put three points on a plane and then connect them to form a triangle. When we move one of the three points, the triangle will change but still remain a triangle. On other case, using an MS Word we draw three line segments and connect them to form a triangle. When we move one of the segments, the figure will change the form at all, not a triangle anymore.

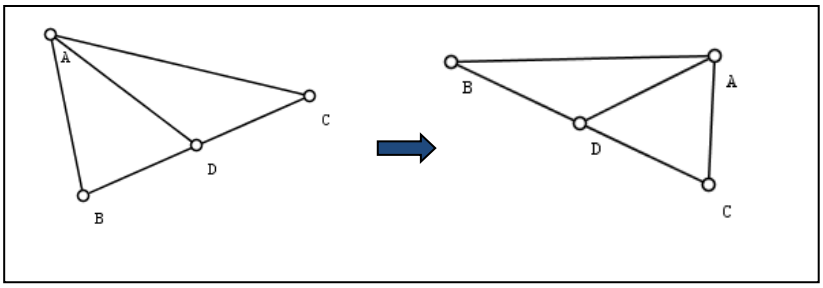


Figure 21 A dynamic geometry
(moving one object will not change the properties of geometrical objects)

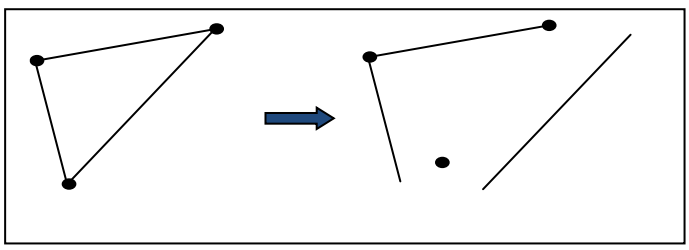


Figure 22 A static geometry (moving one object will change the properties of geometrical objects)

Most DGS allow users to drag geometric objects into different shapes or position without changing their main geometrical properties.

There are many dynamic geometry software. Among the earlier DGS are **Cabri Geometre II+** (www.cabri.com) and **Geometer's Sketchpad** (www.keypress.com/sketchpad). These are two examples of commercial DGS focus on plane (2 dimension) geometry. During the last decade, there have been developing many other commercial and free DGS. Some DGS provide capability for 2D and 3D geometry such as **Cinderella** (www.cinderella.de), **CaR** (http://www.z-u-l.de), and **Geogebra** (www.geogebra.org), some DGS focus more on 3D geometry such as **Cabri 3D** (www.cabri.com) and **Yenka 3D Shapes** (http://yenka.com). Some DGS have similar uses and features, so the next will only focus on the use of GeoGebra, as it is available for free, and also provide many resources for teachers and students (see the Internet resources discussion).

The use of GeoGebra as a tool for mathematics education in secondary schools has becoming more popular during the last decade. The following are some possible ways of using GeoGebra in teaching and learning mathematics.

1) GeoGebra for demonstration and visualisation
   Even in traditional teaching, GeoGebra can be used as a tool for demonstration and visualisation, because GeoGebra is a software with a wide coverage due to its different representations.

2) GeoGebra as a construction tool
   GeoGebra has many features which are importance, as drawing/designing software, for teaching constructive geometry.

3) GeoGebra as an exploration and discovering mathematics tool
   GeoGebra can be used to create dynamic and interactive worksheets for students so they can do explorations and finding certain mathematics concepts, relations, and principles.

4) GeoGebra as an authoring tool for preparing teaching materials
   Teachers can use GeoGebra as a cooperation, communication and representation tool to prepare materials (interactive e-learning content) for teaching and learning mathematics. GeoGebra can be used to create interactive worksheets in two forms: (1) standalone interactive worksheet to open using GeoGebra program, and (2) interactive HTML pages – so called dynamic worksheets - to open using Internet browsers with Java enabled, such as MS Internet, Explorer, Mozilla, or Opera, independently without GeoGebra program to use the worksheet.

The following are some limited examples of the mentioned uses of GeoGebra for teaching and learning mathematics at Junior High School.

1. Interactive System of Linear Equations in Two variables

GeoGebra is very powerful to create interactive worksheets and simulations of mathematical topics, including system of linear equations in two variables. **Figure 23** and Figure **24** show two different GeoGebra worksheets on system of linear equations in two variables. In the first worksheet the linear equations are represented as two lines and the appropriate equation in the form of gradient and y-intercept is displayed around each line. Students can change the values of gradient (m) and y-intercept (b) by moving the dot on the slider. When students change the values, the equations (lines) and the solution (intersection point) will change automatically. Therefore, they can explore some

properties of system of linear equations in two variables such as what happen if 1) the two gradients are the same, 2) all values of b are zeros, and so on.

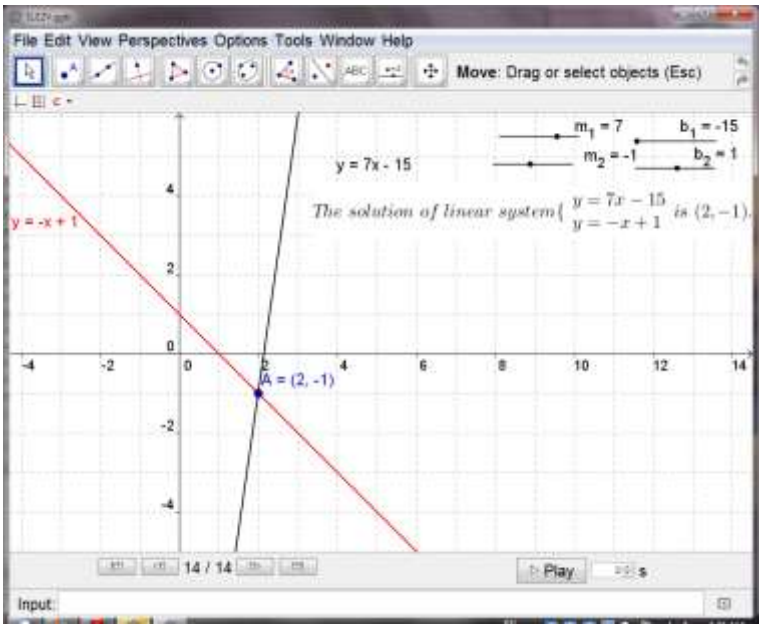


Figure 23 Interactive System of Linear Equations with Two Variables in GeoGebra

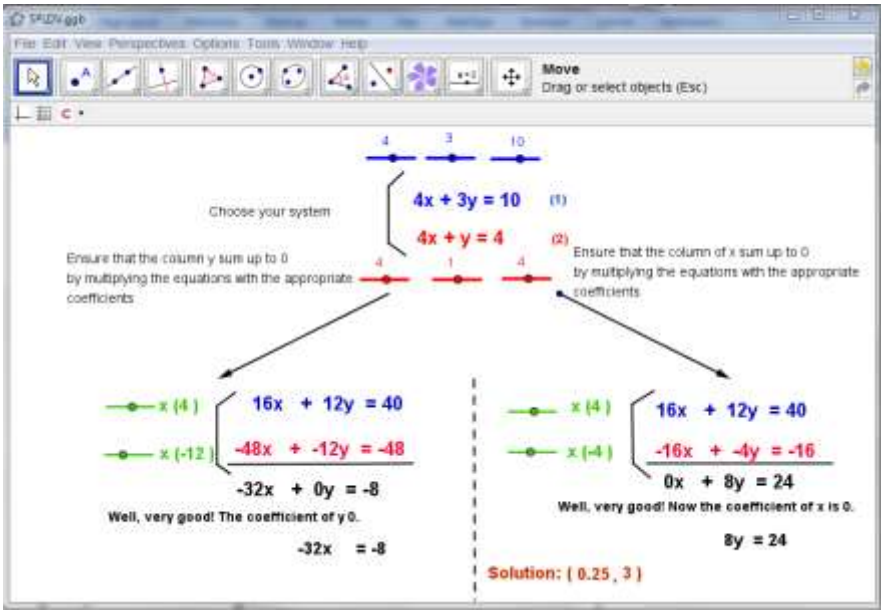


Figure 24 Interactive Ellimination Methods for Solving SLE2V in GeoGebra

In the second worksheet, students can solve a system of linear equations in two variables using an elimination method interactively. First, they may change the equations by changing their coefficients and constants using the sliders. Next, they may eliminate y from both equations by using appropriate multipliers on the left side, and eliminate x from both equations by using appropriate multipliers on the right side. When they have done, the solution will be shown. Otherwise, they have to do the correct elimination processes.

2. Geometry Construction

Geometry construction is a step-by-step process of constructing a geometric figure using only a compass and a ruler, without using a protractor. GeoGebra can be used to show geometry construction processes, such as constructing a triangle from three given line segments, constructing two conqruent triangles, constructing the incircle and circumcircle of a triangle, as shown on the **Figure 25**, **Figure 26**, and **Figure 27**.

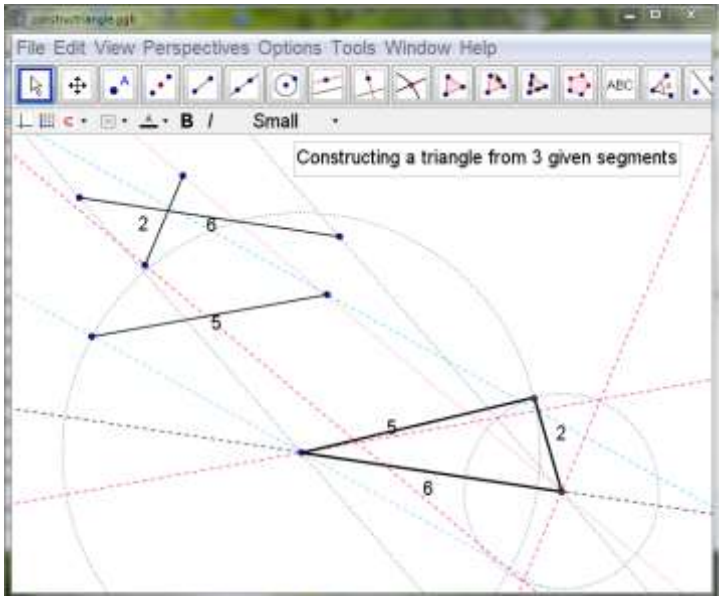


Figure 25 Constructing a Triangle Using Three Given Line Segments

A triangle can be constructed from given three line segments (as far as they satisfy the triangle condition), by constructing equaivalent line segments. Two equivalent line segments can be constructed by using the properties of a parallelogram.

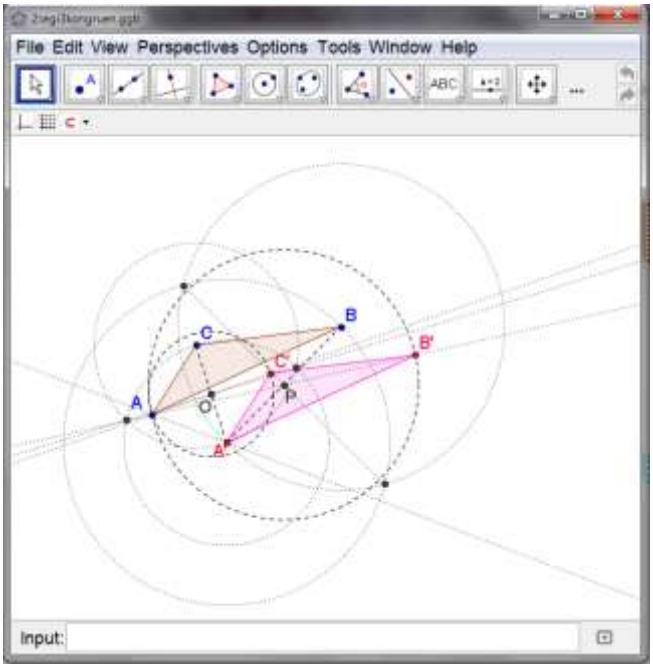


Figure 26 Constructing Two Conqruent Triangles

Constructing two conqruent triangles is a particular case of constructing a triangle from given three line segments.

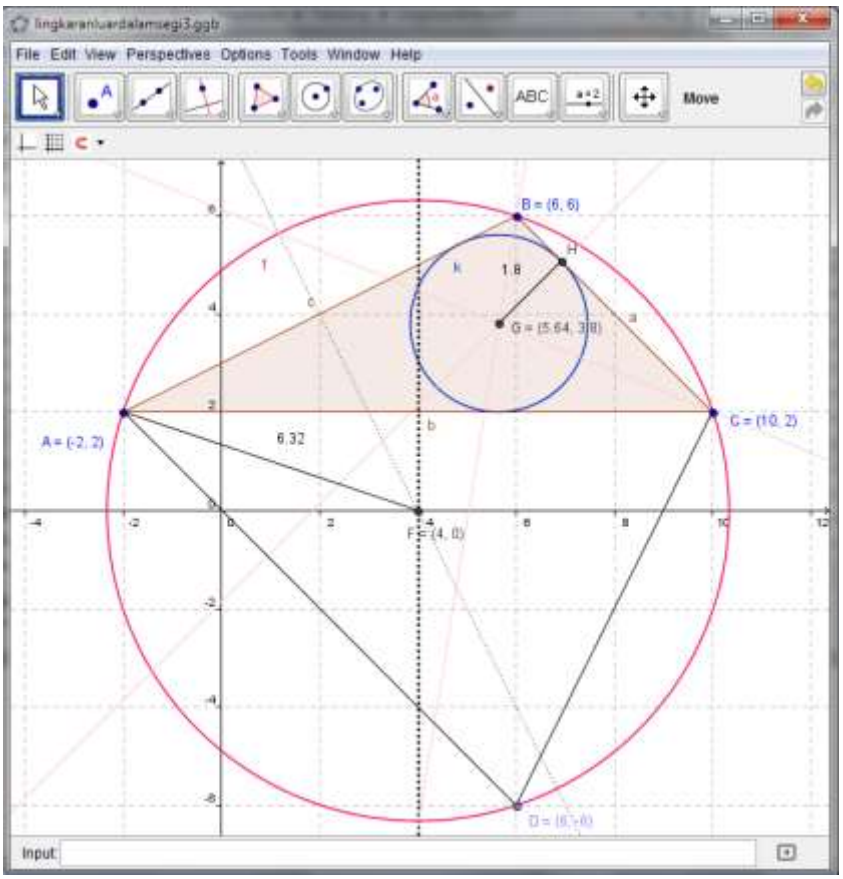


Figure 27 Constructing Inscribed and Circum Circles of a Triangle

An incircle is a circle that is touching each side of a triangle. Its center is the intersection of two angle bisectors. A circumcircle is a circle that is passing through the three vertices of a triangle. Its center is the intersection of two perpendicular bisectors of two sides of the triangle.

Once a student has finished a construction, she/he may change of a line segment by dragging an end point to anywhere and then observe what happen with the construction. She/he may also discover new properties of a geometric figure or a new conjecture.

3. Mathematics Exploration

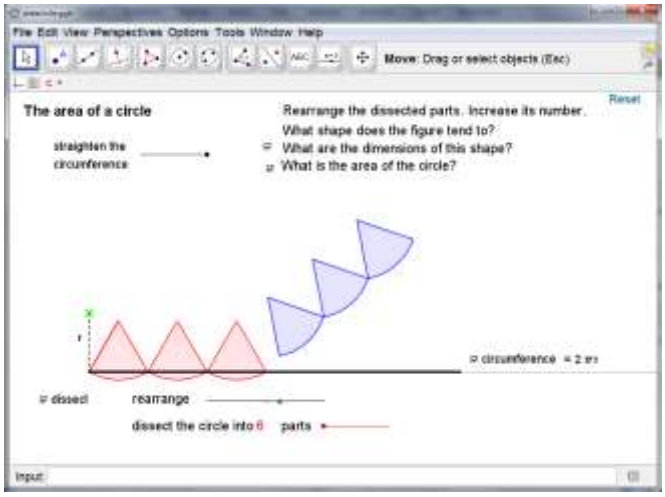


Figure 28 Finding the Area of a Circle

As dynamic mathematics software, GeoGebra can be used to design and to create activities for students to explore some mathematics concepts and principles. By doing these explorations students can go to some conclusions such as finding a formula, a geometric properties, etc. One example is finding the formula of area of a circle (**Figure 28**). Using the interactive GeoGebra worksheet, students can find the formula for the area of circle, by dissecting the circle into a number of parts, then rearranging the parts into parallelogram shape and notifying the dimensions of the shape.

Another example is finding the sum of interior angles of a triangle (**Figure 29**). In the GeoGebra worksheet, students can see the measurements of interior angles of a triangle and its sum. They can also modify the triangle interactively by dragging its vertices to anywhere and observe what happen with its interior angles and its sum. After this exploration they can make a conclusion.

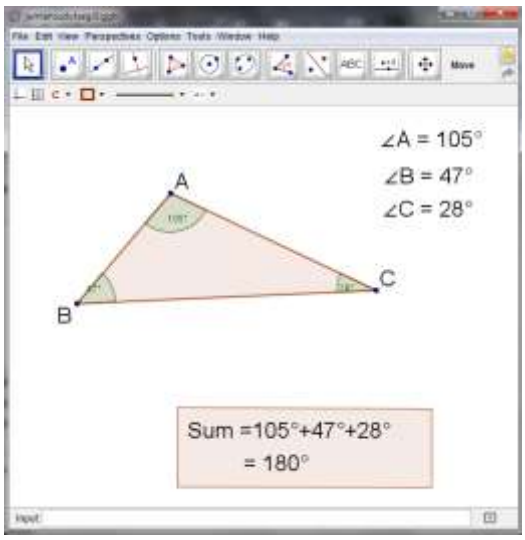


Figure 29 Finding the Sum of Interior Angles of a Triangle

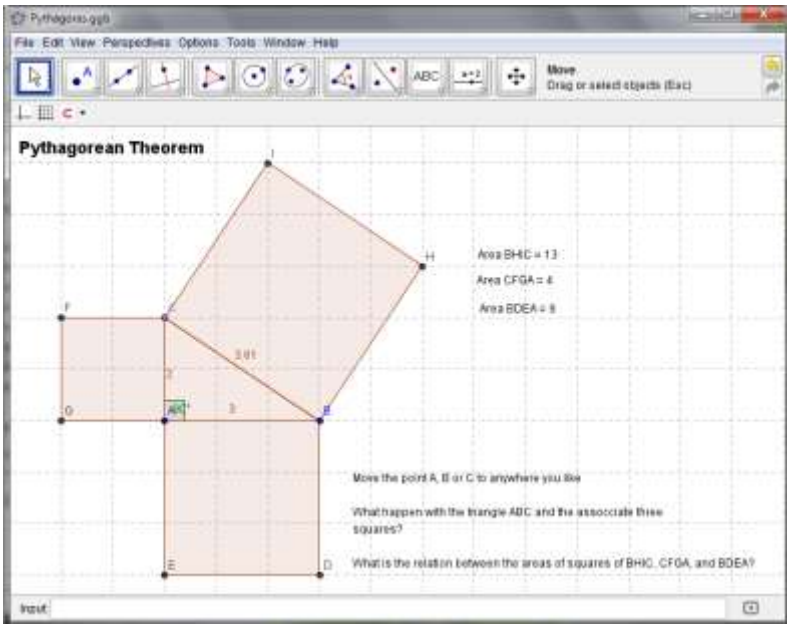


Figure 30 Finding the Theorem of Pythagoras

The next example is exploration on the theorem of Pythagoras (**Figure 30**). In the GeoGebra work-sheet, students can explore the relation between the area of a square on the hypothenusa of a right

triangle and the areas of squares on the right sides. They can change the right triangle into different sizes and observe if the relation still holds. At the end, they have to able to conclude the theorem of Pyhtagoras.

4. Interactive Games

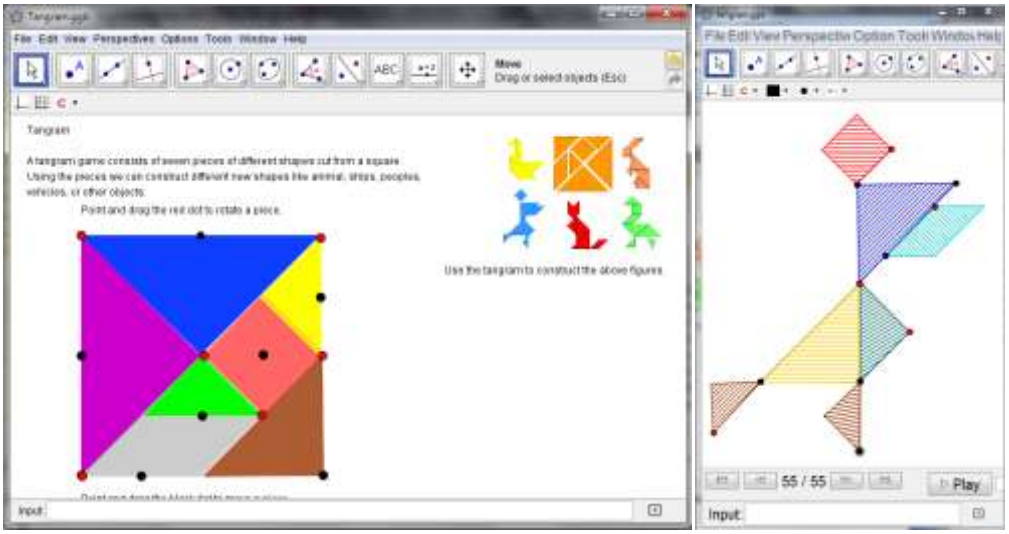


Figure 31 Interactive Tangram

GeoGebra can also be used to create interactive mathematics games and puzzles, like tangram (**Figure 31**) and hexagon-square puzzle (**Figure 32**). Using tangram, students can create different shapes and objects, such as people, animals, ships, vehicles, etc. The hexagon-square puzzle shows how to reconstruct a square from a hexagon and vice-versa. Use the **Zoom** slider to zoom out and zoom in the figure. Use the vertical slider to see the reconsgtruction process. This is a very intersting game that attracting students.

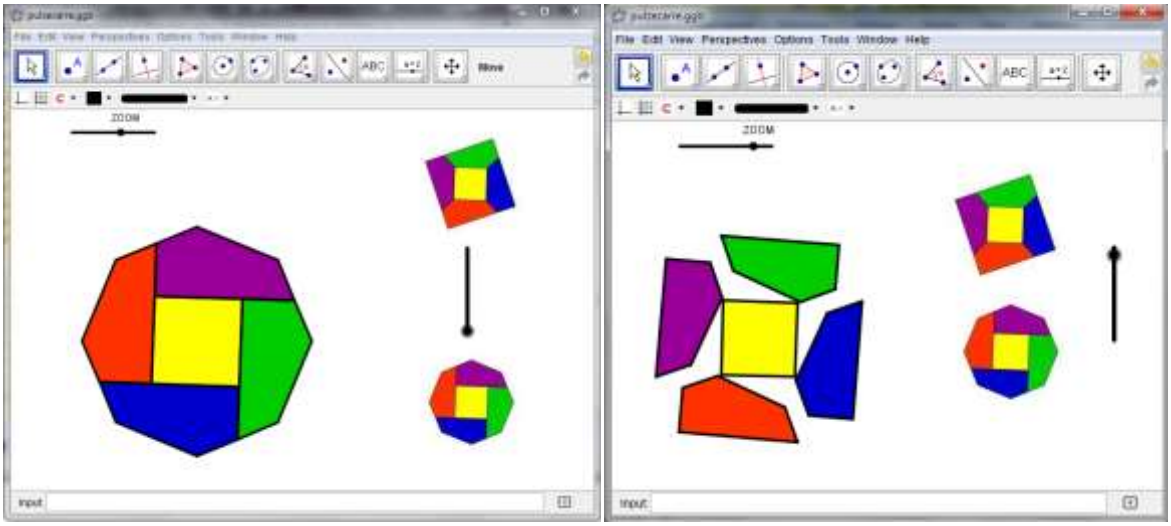


Figure 32 Reconstructing Hexagon into Square

E. Closing Remarks

This paper has described and explored some examples of using ICT to support RME-based teaching and learning for Junior High Schools. However, because of time and space limit, it is not possible to

exhaust all possibilities. Teachers are required to improve their skills in using ICT for supporting students' learning, and to explore more ideas from the Internet.

It is expected that by using the right ICT-based activities in teaching and learning mathematics, the students understanding and competences on mathematics will improve. This, in turn will also improve the students mathematics achievement.

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